AIN SHAMS UNIVERSITY
FACULY OF ENGINEERING

Fluid Mechanics

CEI 121

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Lecturer (18)

C4-2 "The impulse momentum principle"

Application Of The Momentum Equation

STEPS IN ANALYSIS:

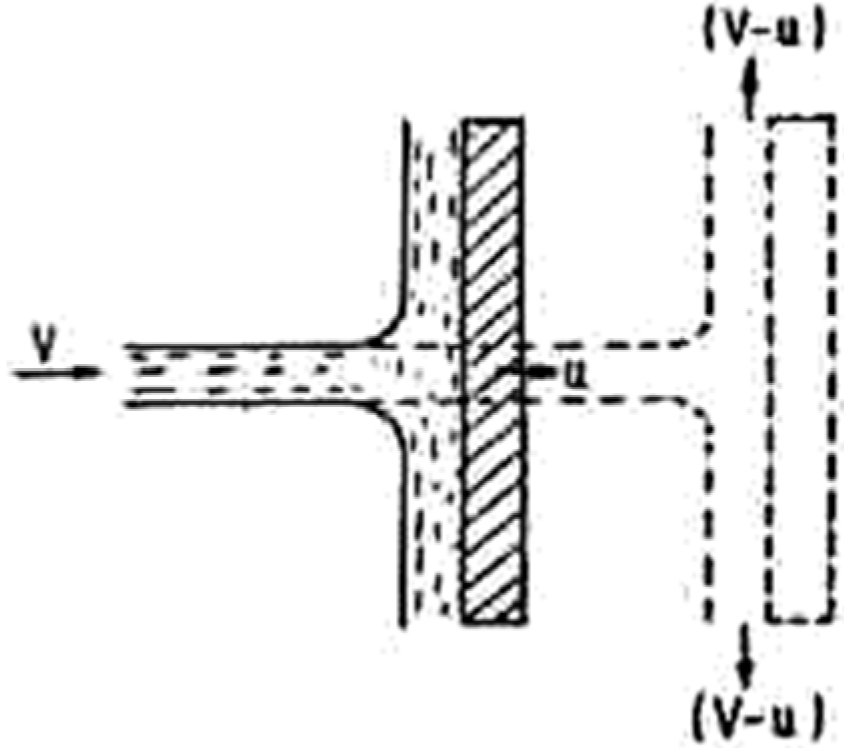
- Draw a control volume
- Decide on co-ordinate axis system
- Calculate the velocity components
- determine all acting forces
- Calculate the force components
- Calculate the resultant force

Example I

A flat plate is struck normally by a jet of water 50mm in diameter with a velocity of 18 m/s. calculate

(a) the force on the plate when it is stationary,

(b) the force on the plate when it moves in the same direction as the jet with a velocity of 6 m/s



- (a) if the plate is stationary:

$$Q = 18 \times \frac{\pi}{4}(0.05)^2 = 0.0353$$

$$F_x = Q\rho(0 - v_1) = -Q\rho v_1$$

$$= -0.0353 \times 10^3 \times 18 = -636.17N$$

$$F_y = 0 \qquad R = -F = 636.17N$$

- (b) if the plate move:

$$Q = (18 - 6) \times \frac{\pi}{4}(0.05)^2 = 0.0235 m^3 / s$$

$$F_x = -Q\rho(v_1 - u)$$

$$= -0.0235 \times 10^3 \times (18 - 6) = -282.4N$$

$$F_y = 0 \qquad R = -F = 282.4N$$

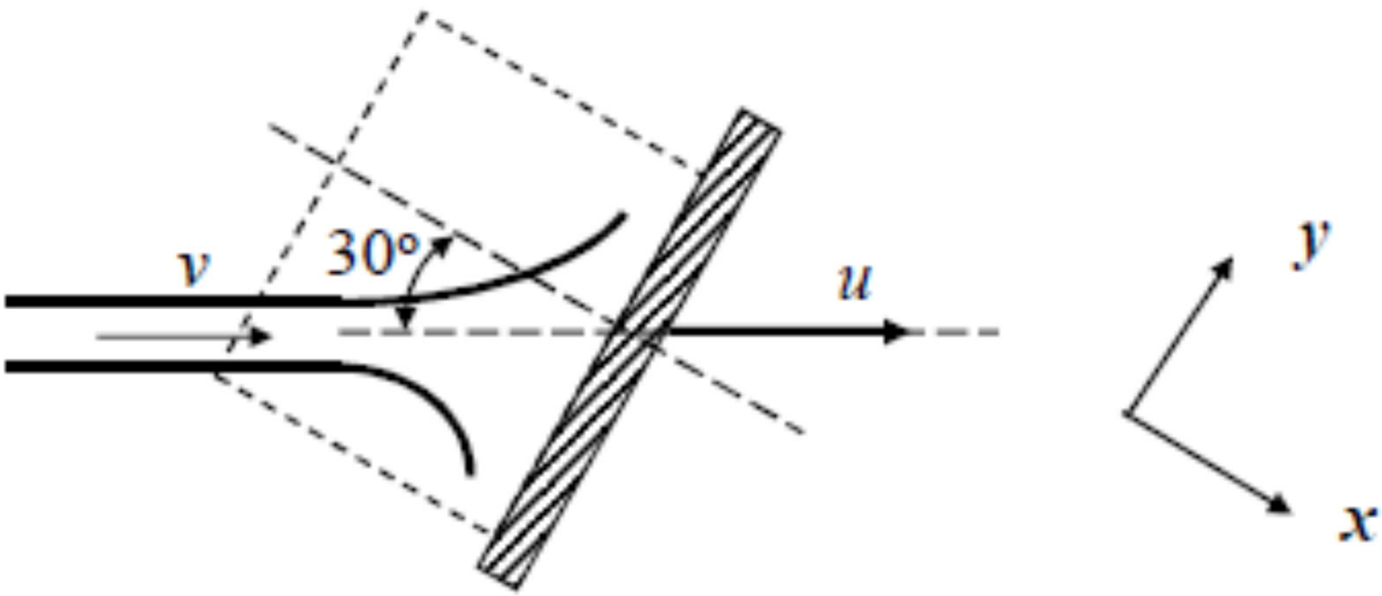
Example II

A jet of water from a fixed nozzle has a diameter d of 25mm and strikes a flat plate at angle u of 30o to the normal to the plate. The velocity of the jet v is 5m/s, and the surface of the plate can be assumed to be frictionless.

Calculate the force exerted normal to the plate

(a) if the plate is stationary.

(b) if the plate is moving with velocity u of 2m/s in the same direction as the jet



(a) if the plate is stationary:

- Initial component of velocity relative to plate in x direction = $v\cos\theta$
- Final component of velocity relative to plate in x direction = 0

$$R_x = \dot{m}(v_{in} - v_{out})_x = \rho A v (v\cos\theta) = \rho A v^2 \cos\theta$$

$$R_x = 1000 \times \left(\frac{\pi}{4} 0.025^2\right) \times 5^2 \times \cos 30 = 10.63 N$$

(b) if the plate move:

Initial component of velocity relative to plate in x direction = $(v-u)\cos\theta$

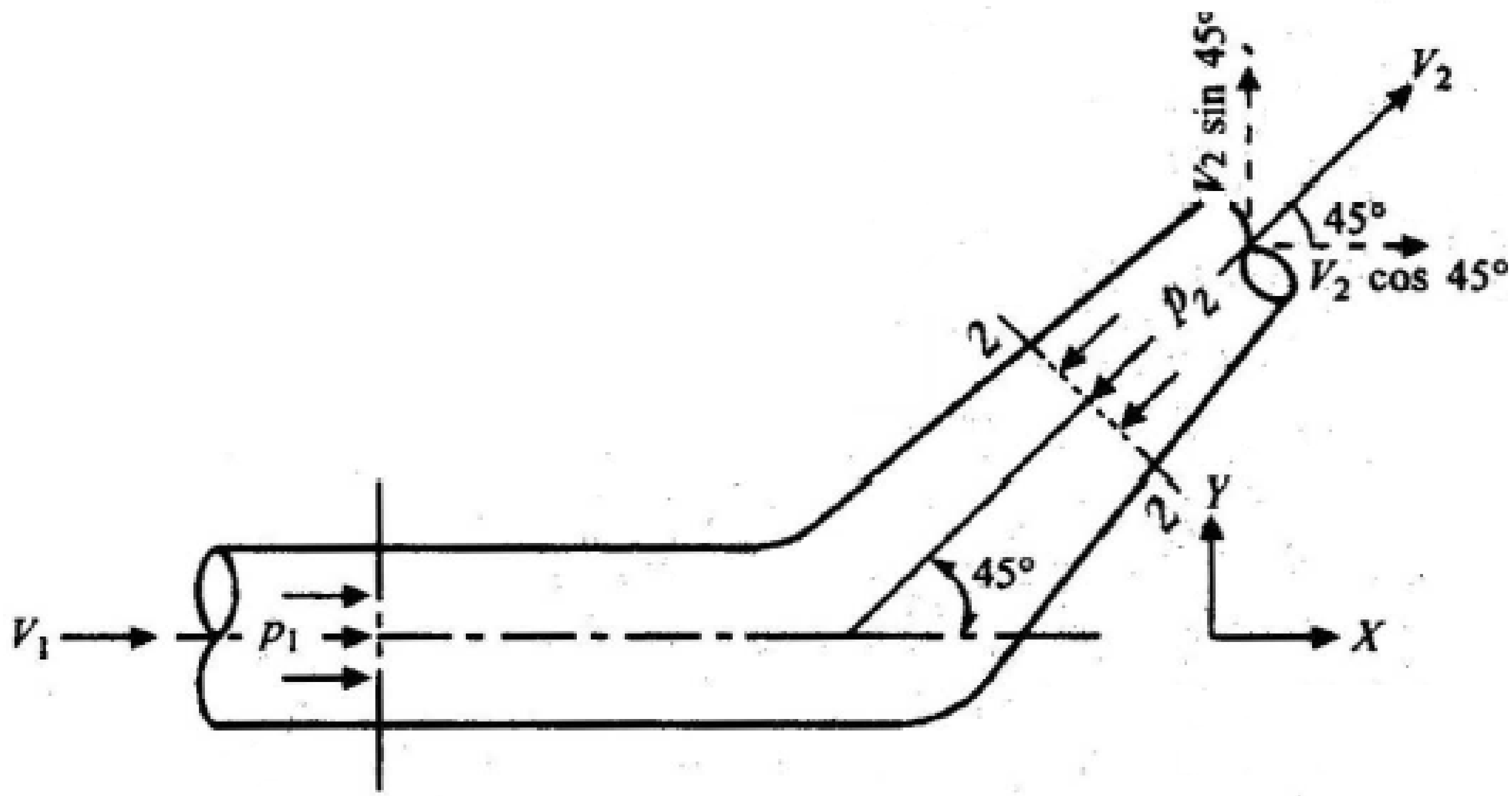
Final component of velocity relative to plate in x direction = 0

$$R_x = \dot{m}(v_{in} - v_{out})_x = \rho A (v-u)(v-u)\cos\theta = \rho A (v-u)^2 \cos\theta$$

$$R_x = 1000 \times \left(\frac{\pi}{4} 0.025^2\right) \times (5-2)^2 \times \cos 30 = 3.83 N$$

Example III

In a 45 degree bend a rectangular air duct of a cross sectional area equals 1 m2 is gradually reduced to 0.5 m2. Find the magnitude and the direction of force required to hold the duct in position if the velocity at the inlet section is 10m/s and the inlet pressure is 30kn/m2 and outlet pressure is 29.82kn/m2

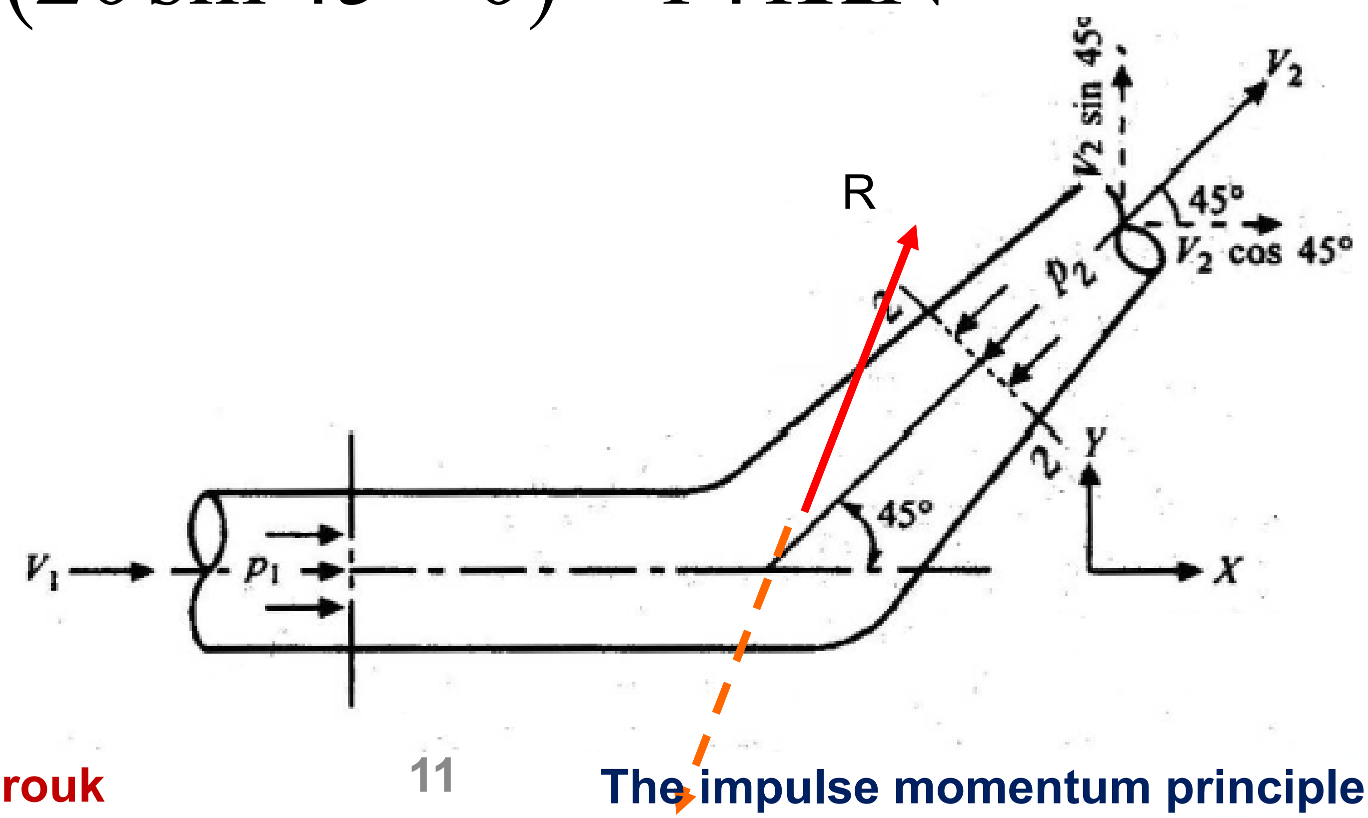


$$\mathbf{F_x} = \mathbf{Q}\rho(\mathbf{v}_2 \cos\theta_2 - \mathbf{v}_1 \cos\theta_1)$$

$$\mathbf{F_x} = 10\times10^3(20\cos 45 - 10) = 41.42\mathbf{KN}$$

$$\mathbf{F_y} = \mathbf{Q}\rho(\mathbf{v}_2 \sin\theta_2 - \mathbf{v}_1 \sin\theta_1)$$

$$\mathbf{F_y} = 10\times10^3(20\sin 45 - 0) = 141\mathbf{KN}$$



$$\mathbf{F_{Px}} = \mathbf{P_1 A_1} \cos\theta_1 + \mathbf{P_2 A_2} \cos\theta_2$$

$$\mathbf{F_{Px}} = 30 \times 1 - 29.82 \times 0.5 \times \cos 45 = 19.46\,\mathbf{KN}$$

$$\mathbf{F_{Py}} = \mathbf{P_1 A_1} \sin\theta_1 + \mathbf{P_2 A_2} \sin\theta_2$$

$$\mathbf{F_{Py}} = 0 - 29.82 \times 0.5 \times \sin 45 = -10.54\,\mathbf{KN}$$

$$\mathbf{F_{Rx}} = \mathbf{F_x} - \mathbf{F_{Px}} = 41.42 - 19.46 = 21.96\,\mathbf{KN}$$

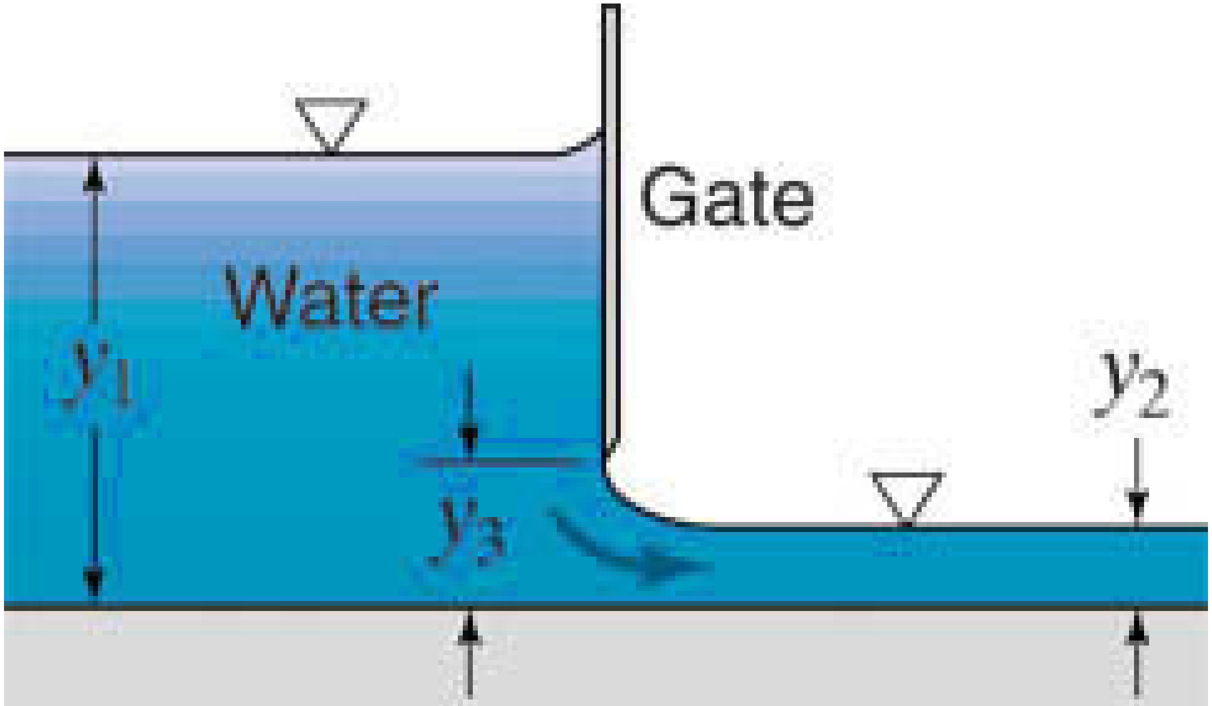
$$\mathbf{F_{Ry}} = \mathbf{F_y} - \mathbf{F_{Py}} = 141.0 - (-10.54) = 151.96\,\mathbf{KN}$$

$$\mathbf{F_R} = \sqrt{(21.96)^2 + (151.96)^2} = 153.54\,\mathbf{KN}$$

$$\phi = \tan^{-1}\left(\frac{151.96}{21.96}\right) = 81.8^\circ$$

Example IV

Find the horizontal thrust of the water on each meter of width of the sluice gate shown in the below figure. Given y1=2.2 m, y2=0.4 m, and y3=0.5 m. Neglect friction.



Let q be the flow rate per meter of width, m^3/s per m.

Continuity, per meter of width:

$$V_1 = q/2.2, \qquad V_2 = q/0.4$$

Energy, neglecting friction:

$$2.2 + (q/2.2)^2/2g = 0.4 + (q/0.4)^2/2g$$

from which $q = 2.42\ m^3/s$ per m width ;

$$V_1 = 1.099\ m/s, \qquad V_2 = 6.04\ m/s$$

$$\Sigma F_x = F_1 - F_2 - F_x = \rho Q(V_2 - V_1)$$

$$9810(2.2^2)/2 - 9810(0.4^2)/2 - F_x = 1000(2.42)(6.04 - 1.099)$$

$$23\,700 - 785 - F_x = 11\,950\ ;$$

$$F_x = +11\,000\ N/m \text{ to the left}$$

Thank you