

SRI VIDYA COLLEGE OF ENGINEERING & TECHNOLOGY
VIRUDHUNAGAR
DEPARTMENT OF CIVIL ENGINEERING

CE6405-MECHANICS OF SOILS

UNIT-I
INTRODUCTION & CLASSIFICATION OF SOILS



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UNIT-I

INTRODUCTION

SOIL AND SOIL ENGINEERING

*** The term Soil has various meanings, depending upon the general field in which it is being considered.**

***To a Pedologist ... Soil is the substance existing on the earth's surface, which grows and develops plant life.**

***To a Geologist Soil is the material in the relative thin surface zone within which roots occur, and all the rest of the crust is grouped under the term ROCK irrespective of its hardness.**

***To an Engineer Soil is the un-aggregated or un-cemented deposits of mineral and/or organic particles or fragments covering large portion of the earth's crust.**

*** Soil Mechanics is one of the youngest disciplines of Civil Engineering involving the study of soil, its behavior and application as an engineering material.**

According to Terzaghi (1948): *"Soil Mechanics is the application of laws of mechanics and hydraulics to engineering problems dealing with sediments and other unconsolidated accumulations of solid particles produced by the mechanical and chemical disintegration of rocks regardless of whether or not they contain an admixture of organic constituent."

*** Geotechnical Engineering Is a broader term for Soil Mechanics.**

*** Geotechnical Engineering contains:**

- Soil Mechanics (Soil Properties and Behavior)
- Soil Dynamics (Dynamic Properties of Soils, Earthquake Engineering, Machine Foundation)
- Foundation Engineering (Deep & Shallow Foundation)
- Pavement Engineering (Flexible & Rigid Pavement)
- Rock Mechanics (Rock Stability and Tunneling)
- Geosynthetics (Soil Improvement)

Soil Formation

* Soil material is the product of rock

* The geological process that produce soil is

WEATHERING (Chemical and Physical).

* Variation in Particle size and shape depends on:

- *Weathering Process*

- *Transportation Process*

* Variation in Soil Structure Depends on:

- *Soil Minerals*

- *Deposition Process*

* *Transportation and Deposition*

What type of soils are usually produced by the different weathering & transportation process

- Boulders

- Gravel Cohesionless

- Sand (Physical)

- Silt Cohesive

- Clay (Chemical)

* These soils can be

- Dry

- Saturated -

Fully - Partially

* Also they have different shapes and textures



SOIL PROPERTIES
PHYSICAL AND INDEX PROPERTIES

1- Soil Composition

- Solids
- Water
- Air

2- Soil Phases

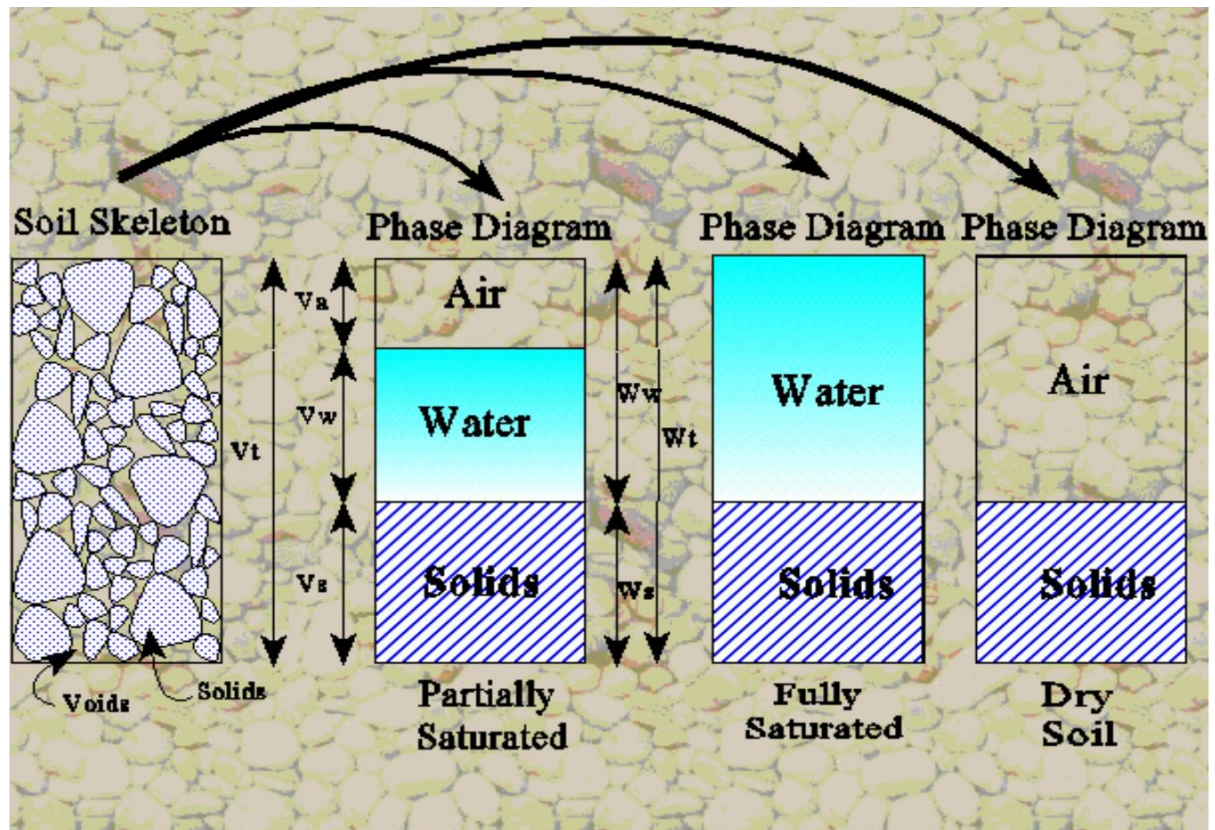
- Dry
- Saturated * Fully Saturated
- * Partially Saturated
- Submerged



3- Analytical Representation of Soil:

For the purpose of defining the physical and index properties of soil it is more convenient to represent the soil skeleton by a block diagram or phase diagram.

4- Weight - Volume Relationships:



Weight

$$W_t = W_w + W_s$$

Volume

$$V_t = V_v + V_s = V_a + V_w + V_s$$

1- Unit Weight - Density

$$\gamma_{soil} = \frac{\text{Total Weight}}{\text{Total Volume}} = \frac{W_t}{V_t}$$

* Also known as

- Bulk Density -

Soil Density

- Unit Weight

- Wet Density

$$3- \text{ Water Content } = W_c = \frac{\text{Weight of Water}}{\text{Weight of Solids}} = \frac{W_w}{W_s} \times 100\%$$

$$4- \text{ Void Ratio } = e = \frac{\text{Volume of Voids}}{\text{Volume of Solids}} = \frac{V_v}{V_s}$$

$$5- \text{ Porosity } = n = \frac{\text{Volume of Voids}}{\text{Total Volume}} = \frac{V_v}{V_t} \times 100\%$$

$$6- \text{ Degree of Saturation } = S_r = \frac{\text{Volume of Water}}{\text{Volume of Voids}} = \frac{V_w}{V_v}$$

$$7- \text{ Relative Density } = D_r = \frac{e_{\max} - e_{\text{field}}}{e_{\max} - e_{\min}} =$$

Relationships Between Basic Properties:

Examples:

Index Properties

Refers to those properties of a soil that indicate the type and conditions of the soil, and provide a relationship to structural properties such as strength, compressibility, permeability, swelling potential, etc.

INDEX PROPERTIES					
PARTICLE SIZE DISTRIBUTION			CONSISTENCY LIMITS		
Mechanical Analysis	Hydrometer Analysis		L.L.	P.L.	P.I
(Coarse Grained Soil)	(Fine Grained Soil)				
Dry Method	Wet Method				

1- PARTICLE SIZE DISTRIBUTION

* It is a screening process in which coarse fractions of soil are separated by means of series of sieves.

* Particle sizes larger than 0.074 mm (U.S. No. 200 sieve) are usually analyzed by means of sieving. Soil materials finer than 0.074 mm (-200 material) are analyzed by means of sedimentation of soil particles by gravity (hydrometer analysis).

1-1 MECHANICAL METHOD

U.S. Standard Sieve:

Sieve No. 4 10 20 40 60 100 140 200 280

Opening in mm 4.76 2.00 0.84 0.42 0.25 0.15 0.105 0.074 -

Cumulative Curve:

* *A linear scale is not convenient to use to size all the soil particles (opening from 200 mm to 0.002 mm).*

* *Logarithmic Scale is usually used to draw the relationship between the % Passing and the Particle size.*

Example:

Parameters Obtained From Grain Size Distribution Curve:

1- Uniformity Coefficient C_u (measure of the particle size range)

C_u is also called Hazen Coefficient

$$C_u = D_{60} / D_{10}$$

$C_u < 5$ ----- Very Uniform

$C_u = 5$ ----- Medium Uniform

$C_u > 5$ ----- Nonuniform

2- Coefficient of Gradation or Coefficient of Curvature C_g

(measure of the shape of the particle size curve) $C_g = (D_{30})^2 /$

$D_{60} \times D_{10}$

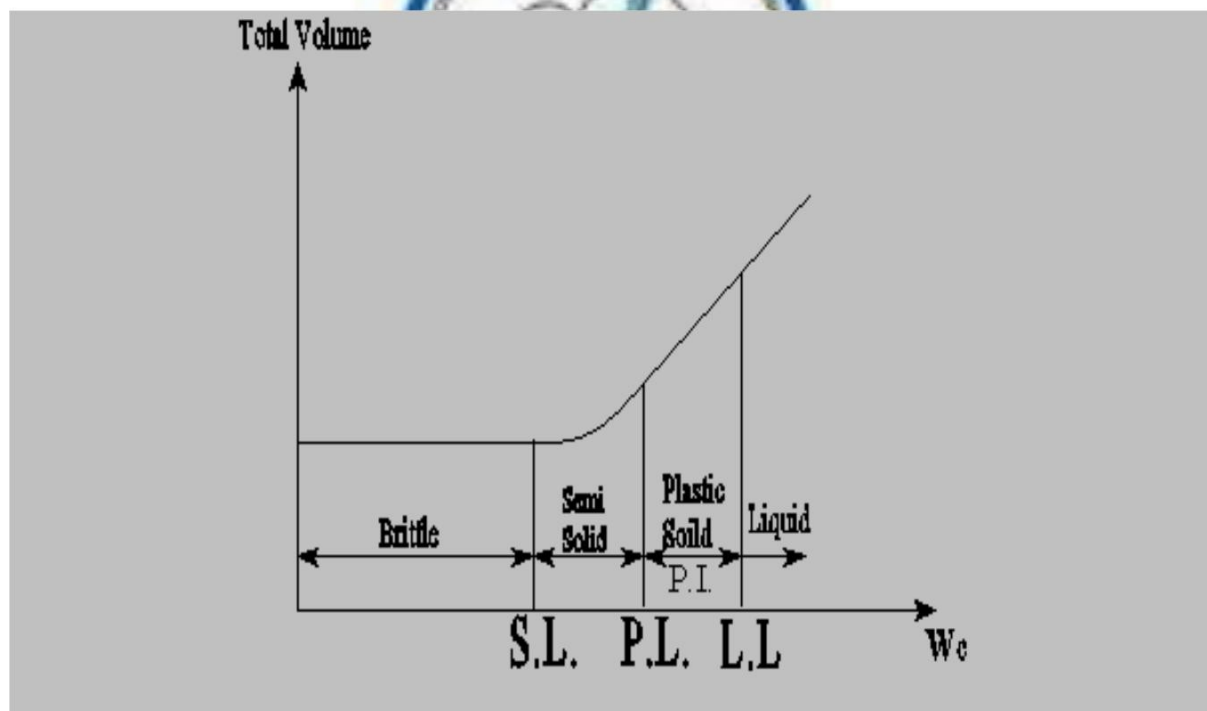
C_g from 1 to 3 ----- well graded

3- Coefficient of Permeability

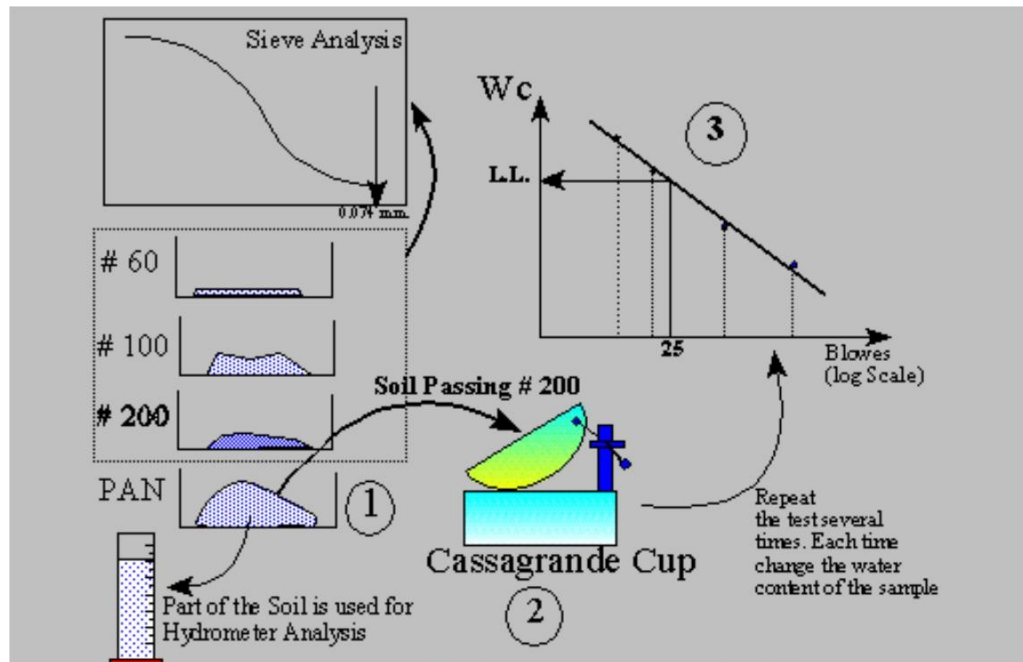
$k = C_k (D_{10})^2 \text{ m/sec}$

Consistency Limits or Atterberg Limits:

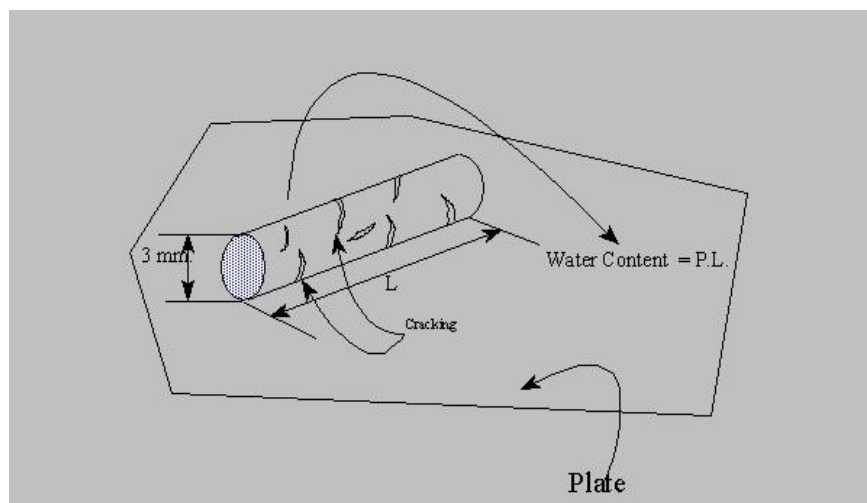
- State of Consistency of cohesive soil



1- Determination of Liquid Limit:



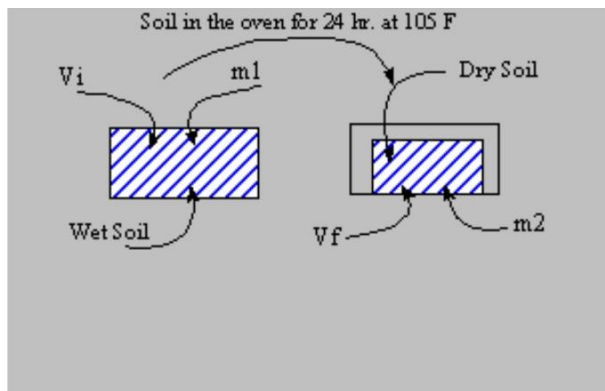
2- Determination of Plastic Limit:



3- Determination of Plasticity Index

$$P.I. = L.L. - P.L.$$

4- Determination of Shrinkage Limit



$$L. = \left(\frac{m_1 - m_2}{m_2} \right) (100) \cdot \left[\frac{(V_i - V_f) \rho_w}{m_2} \right] (100)$$

5- Liquidity Index:

$$LI = \frac{w_c - P.L.}{L.L. - P.L.}$$

6- Activity:

$$A = \frac{P.I.}{\% \text{ Clay Fraction (weight)}}$$

CLASSIFICATION OF SOIL

Classification of soil is the separation of soil into classes or groups each having similar characteristics and potentially similar behaviour. A classification for engineering purposes should be based mainly on mechanical properties: permeability, stiffness, strength. The class to which a soil belongs can be used in its description.

The aim of a classification system is to establish a set of conditions which will allow useful comparisons to be made between different soils. The system must be simple. The relevant criteria for classifying soils are the **size distribution** of particles and the **plasticity** of the soil.

Particle Size Distribution

For measuring the distribution of particle sizes in a soil sample, it is necessary to conduct different **particle-size tests**.

Wet sieving is carried out for separating fine grains from coarse grains by washing the soil specimen on a 75 micron sieve mesh.

Dry sieve analysis is carried out on particles coarser than 75 micron. Samples (with fines removed) are dried and shaken through a set of sieves of descending size. The weight retained in each sieve is measured. The cumulative percentage quantities finer than the sieve sizes (passing each given sieve size) are then determined.

The resulting data is presented as a distribution curve with **grain size** along x-axis (log scale) and **percentage passing** along y-axis (arithmetic scale).

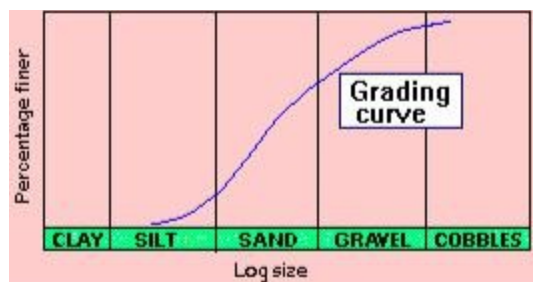
Sedimentation analysis is used only for the soil fraction finer than 75 microns. Soil particles are allowed to settle from a suspension. The decreasing density of the suspension is measured at various time intervals. The procedure is based on the principle that in a suspension, the terminal velocity of a spherical particle is governed by the diameter of the particle and the properties of the suspension.

In this method, the soil is placed as a suspension in a jar filled with distilled water to which a deflocculating agent is added. The soil particles are then allowed to settle down. The concentration of particles remaining in the suspension at a particular level can be determined by using a hydrometer. Specific gravity readings of the solution at that same level at different time intervals provide information about the size of particles that have settled down and the mass of soil remaining in solution.

The results are then plotted between **% finer (passing)** and **log size**.

Grain-Size Distribution Curve

The size distribution curves, as obtained from coarse and fine grained portions, can be combined to form one complete **grain-size distribution curve** (also known as **grading curve**). A typical grading curve is shown.

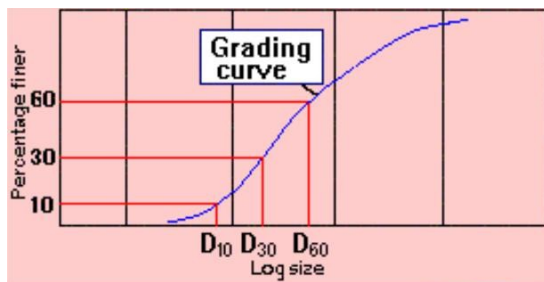


From the complete grain-size distribution curve, useful information can be obtained such as:

1. **Grading characteristics**, which indicate the uniformity and range in grain-size distribution.
2. **Percentages (or fractions)** of gravel, sand, silt and clay-size.

Grading Characteristics

A grading curve is a useful aid to soil description. The geometric properties of a grading curve are called **grading characteristics**.



To obtain the grading characteristics, three points are located first on the grading curve.

D_{60} = size at 60% finer by weight

D_{30} = size at 30% finer by weight

D_{10} = size at 10% finer by weight

The grading characteristics are then determined as follows:

1. **Effective size** = D_{10}

2. **Uniformity coefficient**,

$$C_u = \frac{D_{60}}{D_{10}}$$

3. **Curvature coefficient**,

$$C_c = \frac{(D_{30})^2}{D_{60} \cdot D_{10}}$$

Both C_u and C_c will be 1 for a single-sized soil.

$C_u > 5$ indicates a **well-graded soil**, i.e. a soil which has a distribution of particles over a wide size range.

C_c **between 1 and 3** also indicates a well-graded soil.

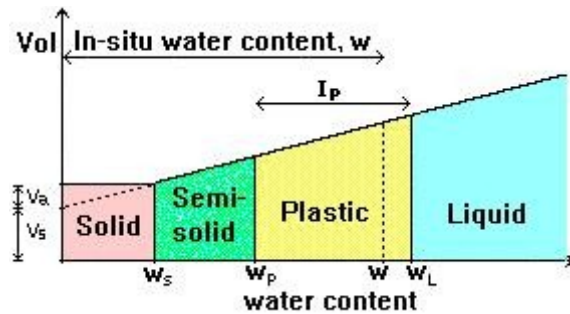
$C_u < 3$ indicates a **uniform soil**, i.e. a soil which has a very narrow particle size range.

Consistency of Soils

The **consistency** of a fine-grained soil refers to its firmness, and it varies with the water content of the soil.

A gradual increase in water content causes the soil to change from **solid** to **semi-solid** to **plastic** to **liquid** states. The water contents at which the consistency changes from one state to the other are called **consistency limits** (or **Atterberg limits**).

The three limits are known as the shrinkage limit (W_s), plastic limit (W_p), and liquid limit (W_L) as shown. The values of these limits can be obtained from laboratory tests.



Two of these are utilised in the classification of fine soils:

Liquid limit (W_L) - change of consistency from plastic to liquid state

Plastic limit (W_p) - change of consistency from brittle/crumbly to plastic state

The difference between the liquid limit and the plastic limit is known as the **plasticity index** (I_p), and it is in this range of water content that the soil has a plastic consistency. The consistency of most soils in the field will be plastic or semi-solid.

Classification Based on Grain Size

The range of particle sizes encountered in soils is very large: from boulders with dimension of over 300 mm down to clay particles that are less than 0.002 mm. Some clays contain particles less than 0.001 mm in size which behave as colloids, i.e. do not settle in water.

In the **Indian Standard Soil Classification System (ISSCS)**, soils are classified into groups according to size, and the groups are further divided into coarse, medium and fine sub-groups.

The grain-size range is used as the basis for grouping soil particles into boulder, cobble, gravel, sand, silt or clay.

Very coarse soils	Boulder size		> 300 mm
	Cobble size		80 - 300 mm
Coarse soils	Gravel size (G)	Coarse	20 - 80 mm
		Fine	4.75 - 20 mm
	Sand size (S)	Coarse	2 - 4.75 mm
		Medium	0.425 - 2 mm
		Fine	0.075 - 0.425 mm
Fine soils	Silt size (M)		0.002 - 0.075 mm
	Clay size (C)		< 0.002 mm

Gravel, sand, silt, and clay are represented by **group symbols G, S, M, and C** respectively.

Physical weathering produces very coarse and coarse soils. Chemical weathering produce generally fine soils.

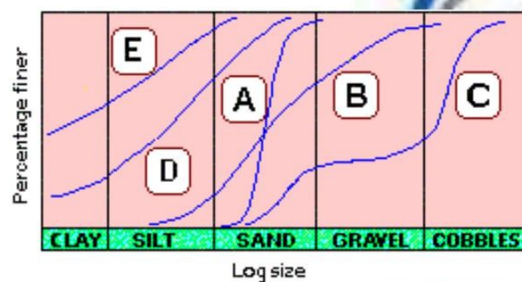
Indian Standard Soil Classification System

Coarse-grained soils are those for which more than 50% of the soil material by weight has particle sizes greater than 0.075 mm. They are basically divided into either gravels (G) or sands (S).

According to **gradation**, they are further grouped as well-graded (**W**) or poorly graded (**P**). If **fine soils** are present, they are grouped as containing silt fines (**M**) or as containing clay fines (**C**).

For example, the combined symbol **SW** refers to well-graded sand with no fines.

Both the position and the shape of the grading curve for a soil can aid in establishing its identity and description. Some typical grading curves are shown.



Curve A - a poorly-graded medium SAND

Curve B - a well-graded GRAVEL-SAND (i.e. having equal amounts of gravel and sand)

Curve C - a gap-graded COBBLES-SAND

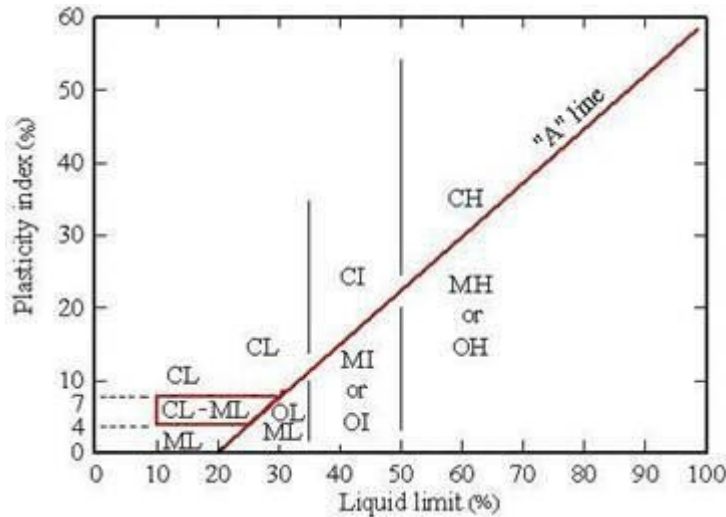
Curve D - a sandy SILT

Curve E - a silty CLAY (i.e. having little amount of sand)

Indian Standard Soil Classification System

Fine-grained soils are those for which more than 50% of the material has particle sizes less than 0.075 mm. Clay particles have a **flaky** shape to which water adheres, thus imparting the property of **plasticity**.

A **plasticity chart**, based on the values of liquid limit (**W_L**) and plasticity index (**I_P**), is provided in **ISSCS** to aid classification. The '**A**' line in this chart is expressed as **$I_P = 0.73 (W_L - 20)$** .



Depending on the point in the chart, fine soils are divided into **clays (C)**, **silts (M)**, or **organic soils (O)**. The organic content is expressed as a percentage of the mass of organic matter in a given mass of soil to the mass of the dry soil solids. Three divisions of plasticity are also defined as follows.

Low plasticity	$W_L < 35\%$
Intermediate plasticity	$35\% < W_L < 50\%$
High plasticity	$W_L > 50\%$

The 'A' line and vertical lines at W_L equal to 35% and 50% separate the soils into various classes.

For example, the combined symbol **CH** refers to clay of high plasticity.

Indian Standard Soil Classification System

Soil classification using group symbols is as follows:

Group Symbol	Classification
Coarse soils	
GW	Well-graded GRAVEL
GP	Poorly-graded GRAVEL
GM	Silty GRAVEL
GC	Clayey GRAVEL
SW	Well-graded SAND
SP	Poorly-graded SAND
SM	Silty SAND
SC	Clayey SAND

Fine soils	
ML	SILT of low plasticity
MI	SILT of intermediate plasticity
MH	SILT of high plasticity
CL	CLAY of low plasticity
CI	CLAY of intermediate plasticity
CH	CLAY of high plasticity
OL	Organic soil of low plasticity
OI	Organic soil of intermediate plasticity
OH	Organic soil of high plasticity
Pt	Peat

Indian Standard Soil Classification System

Activity

"Clayey soils" necessarily do not consist of 100% clay size particles. The proportion of clay mineral flakes (< 0.002 mm size) in a fine soil increases its tendency to swell and shrink with changes in water content. This is called the **activity** of the clayey soil, and it represents the degree of plasticity related to the clay content.

Activity = (Plasticity index) / (% clay particles by weight)

Classification as per activity is:

Activity	Classification
< 0.75	Inactive
0.75 - 1.25	Normal
> 1.25	Active

Liquidity Index

In fine soils, especially with clay size content, the existing state is dependent on the current water content (**w**) with respect to the consistency limits (or Atterberg limits). The **liquidity index (LI)** provides a quantitative measure of the present state.

$$LI = \frac{w - W_p}{I_p}$$

Classification as per liquidity index is:

Liquidity index	Classification
> 1	Liquid
0.75 - 1.00	Very soft
0.50 - 0.75	Soft
0.25 - 0.50	Medium stiff
0 - 0.25	Stiff
< 0	Semi-solid

Visual Classification

Soils possess a number of physical characteristics which can be used as aids to identification in the field. A handful of soil rubbed through the fingers can yield the following:

SAND (and coarser) particles are visible to the naked eye.

SILT particles become dusty when dry and are easily brushed off hands.

CLAY particles are sticky when wet and hard when dry, and have to be scraped or washed off hands.

Worked Example

The following test results were obtained for a fine-grained soil:

$$W_L = 48\% ; W_P = 26\%$$

$$\text{Clay content} = 55\%$$

$$\text{Silt content} = 35\%$$

$$\text{Sand content} = 10\%$$

$$\text{In situ moisture content} = 39\% = w$$

Classify the soil, and determine its activity and liquidity index

Solution:

$$\text{Plasticity index, } I_P = W_L - W_P = 48 - 26 = 22\%$$

Liquid limit lies between 35% and 50%.

According to the Plasticity Chart, the soil is classified as CI, i.e. clay of intermediate plasticity.

$$\Rightarrow \text{Activity} = \frac{I_P}{\text{Clay content}} = \frac{22}{25} = 0.88$$

$$\text{Liquidity index, } \frac{LL}{I_p} = \frac{W - W_p}{I_p} = \frac{39 - 26}{22} = 0.59$$

The clay is of normal activity and is of soft consistency.

SOIL CLASSIFICATION SYSTEMS

*** Why do we need to classify soils ????????????**

To describe various soil types encountered in the nature in a systematic way and gathering soils that have distinct physical properties in groups and units.

*** General Requirements of a soil Classification System: 1-**

Based on a scientific method

2- Simple

3- Permit classification by visual and manual tests.

4- Describe certain engineering properties

5- Should be accepted to all engineers

*** Various Soil Classification Systems:**

1- Geologic Soil Classification System

2- Agronomic Soil Classification System

3- Textural Soil Classification System (USDA)

4- American Association of State Highway Transportation Officials System (AASHTO)

5- Unified Soil Classification System (USCS)

6- American Society for Testing and Materials System (ASTM)

7- Federal Aviation Agency System (FAA)

8- Others

1- Unified Soil Classification (USC) System:

The main Groups:

G = Gravel



S = Sand

.....

M = Silt

C = Clay

O = Organic

*** For Cohesionless Soil (Gravel and Sand), the soil can be Poorly Graded or Well Graded**

Poorly Graded = P

Well Graded = W

*** For Cohesive Soil (Silt & Clay), the soil can be Low Plastic or High Plastic**

Low Plastic = L

High Plastic = H

Therefore, we can have several combinations of soils such as: GW

= Well Graded Gravel

GP = Poorly Graded Gravel

GM = Silty Gravel

GC = Clayey Gravel



Passing Sieve # 4

SW = Well Graded Sand

SP = Poorly Graded Sand

SM = Silty Sand

SC = Clayey Sand

Passing Sieve # 200

ML = Low Plastic Silt

CL = Low Plastic Clay

MH = High Plastic Silt

CH = High Plastic Clay

To conclude if the soil is low plastic or high plastic use Gassagrande's Chart

2- American Association of State Highway Transportation Officials System (AASHTO):

- Soils are classified into 7 major groups A-1 to A-7

Granular A-1 {A-1-a - A-1-b}

(Gravel & Sand) A-2 {A-2-4 - A-2-5 - A-2-6 - A-2-6} A-3

More than 35% pass #

200 A-4

Fine A-5

(Silt & Clay) A-6

A-7



Group Index:

$$I_p + 0.005 (L.L. - 40) + 0.01$$

3- Textural Soil Classification System (USDA)

**** USDA considers only:***

Sand

Silt

Clay

No. Gravel in the System

** If you encounter gravel in the soil ----- Subtract the % of gravel from the 100%.*

** 12 Subgroups in the system*

MOISTURE DENSITY RELATIONSHIPS

(SOIL COMPACTION)

INTRODUCTION:

** In the construction of highway embankments, earth dams, and many other engineering projects, loose soils must be compacted to increase their unit weight.*

** Compaction improves characteristics of soils:*

1- Increases Strength

2- Decreases permeability

3- Reduces settlement of foundation

4- Increases slope stability of embankments

** Soil Compaction can be achieved either by static or dynamic loading: 1-*

Smooth-wheel rollers

2- Sheepfoot rollers

3- Rubber-tired rollers

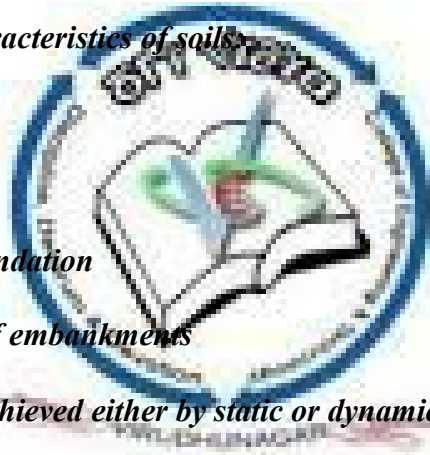
4- Vibratory Rollers

5- Vibroflotation

General Principles:

** The degree of compaction of soil is measured by its unit weight, γ_{dm} , and optimum moisture content, w_c .*

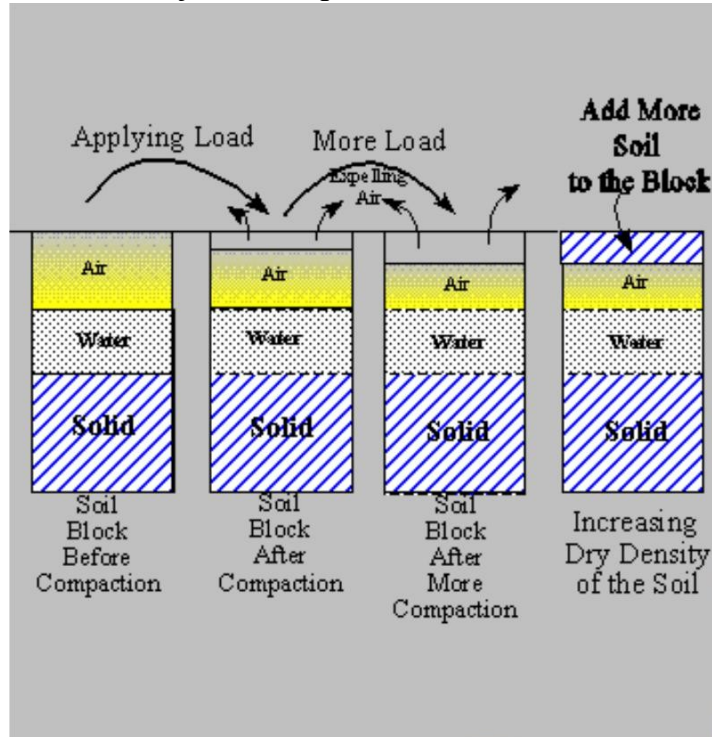
** The process of soil compaction is simply expelling the air from the voids.*



or reducing air voids

* Reducing the water from the voids means consolidation.

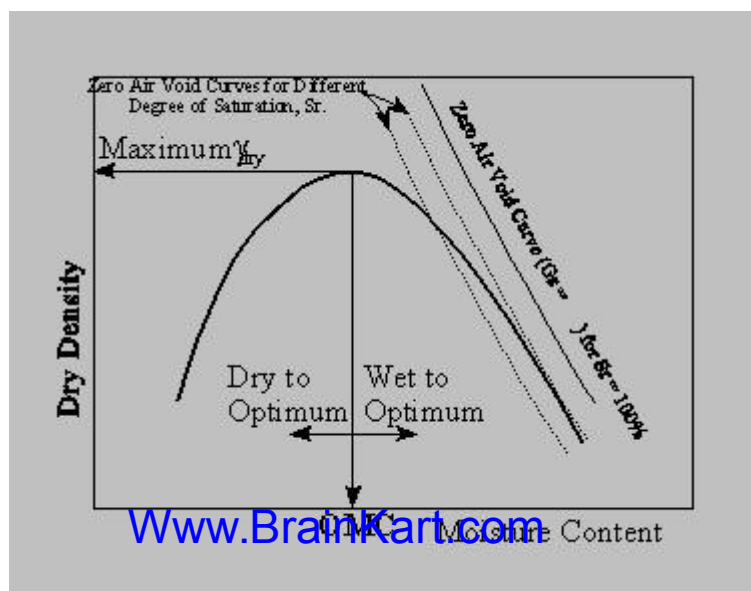
Mechanism of Soil Compaction:



* By reducing the air voids, more soil can be added to the block. When moisture is added to the block (water content, w_c , is increasing) the soil particles will slip more on each other causing more reduction in the total volume, which will result in adding more soil and, hence, the dry density γ_{dry} will increase, accordingly.

* Increasing W_c will increase γ_{dry}

Up to a certain limit (Optimum moisture Content, OMC) After this limit



Increasing W_c will decrease

γ_{dry}

Density-Moisture Relationship

Knowing the wet unit weight and the moisture content, the dry unit weight can be determined from:

$$\gamma_{dry} = \frac{\gamma_{wet}}{1 + \frac{w_c(\%)}{100}}$$

The theoretical maximum dry unit weight assuming zero air voids is:

$$\gamma_{zav} = \frac{G_s \gamma_w}{1 + \frac{w_c G_s}{S_r}} = \frac{\gamma_w}{\frac{w_c}{S_r} + \frac{1}{G_s}}$$

I- Laboratory Compaction:

** Two Tests are usually performed in the laboratory to determine the maximum dry unit weight and the OMC.*

1- Standard Proctor Test

2- Modified Proctor Test

In both tests the compaction energy is:

$$E = \frac{\text{Number of blows per layer} \times \text{Number of layers} \times \text{Weight of hammer} \times \text{Height of drop of hammer}}{\text{volume of mold}}$$

1- Standard Proctor Test

Factors Affecting Compaction:

1- Effect of Soil Type

2- Effect of Energy on Compaction

3- Effect of Compaction on Soil Structure

4- Effect of Compaction on Cohesive Soil Properties

Field Compaction Equipment

There is a wide range of compaction equipment. The compaction achieved will depend on the thickness of lift (or layer), the type of roller, the no. of passes of the roller, and the intensity of pressure on the soil. The selection of equipment depends on the soil type as indicated.



Equipment	Most suitable soils	Least suitable soils
Smooth steel drum rollers (static or vibratory)	Well-graded sand-gravel, crushed rock, asphalt	Uniform sands, silty sands, soft clays
Pneumatic tyred rollers	Most coarse and fine soils	Very soft clays
Sheepsfoot rollers	Fine grained soils, sands and gravels with > 20% fines	Uniform gravels, very coarse soils
Grid rollers	Weathered rock, well-graded coarse soils	Uniform materials, silty clays, clays
Vibrating plates	Coarse soils with 4 to 8% fines	
Tampers and rammers	All soil types	

II- Field Compaction- Control and Specifications

Control Parameter

Dry density and water content correlate well with the engineering properties, and thus they are convenient construction control parameters. Since the objective of compaction is to stabilize soils and improve their engineering behavior, it is important to keep in mind the desired engineering properties of the fill, not just its dry density and water content. This point is often lost in the earthwork construction control.

Design-Construct Procedures

- **Laboratory tests are conducted on samples of the proposed borrow materials to define the properties required for design.**
- **After the earth structure is designed, the compaction specifications are written. Field compaction *control tests* are specified, and the results of these become the standard for controlling the project.**

Specifications

(1) End-product specifications

This specification is used for most highways and building foundation, as long as the contractor is able to obtain the specified *relative compaction*, how he obtains it doesn't matter, nor does the equipment he uses.

Care the results only !

(2) Method specifications

The type and weight of roller, the number of passes of that roller, as well as the lift thickness are specified. A maximum allowable size of material may also be specified.

It is typically used for large compaction project

Relative Compaction (R.C.)

Determine the Relative Compaction in the Field

Where and When

- **First, the test site is selected. It should be representative or typical of the compacted lift and borrow material. Typical specifications call for a new field test for every 1000 to 3000 m² or so, or when the borrow material changes significantly. It is also advisable to make the field test at least one or maybe two compacted lifts below the already compacted ground surface, especially when sheepsfoot rollers are used or in granular soils.**

Method

- **Field control tests, measuring the dry density and water content in the field can either be *destructive* or *nondestructive*.**

Destructive Methods

(a) Sand cone

(b) Balloon

(c) Oil (or water)

method Calculations

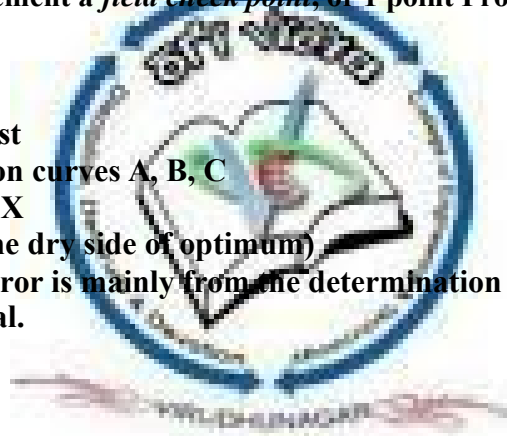
- Know M_s and V_t
- Get ρ_d field and w (water content)
- Compare ρ_d field with ρ_d max-lab and calculate relative compaction R.C.

Destructive Methods

Sometimes, the laboratory maximum density may not be known exactly. It is not uncommon, especially in highway construction, for a series of laboratory compaction tests to be conducted on “representative” samples of the borrow materials for the highway. If the soils at the site are highly varied, there will be no laboratory results to be compared with. It is time consuming and expensive to conduct a new compaction curve. The alternative is to implement a *field check point*, or 1 point Proctor test.

Check Point Method

- 1 point Proctor test
- Known compaction curves A, B, C
- Field check point X (it should be on the dry side of optimum)
- The measuring error is mainly from the determination of the volume of the excavated material.



For example,

- For the sand cone method, the vibration from nearby working equipment will increase the density of the sand in the hole, which will give a larger hole volume and a lower field density.
- If the compacted fill is gravel or contains large gravel particles. Any kind of unevenness in the walls of the hole causes a significant error in the balloon method.
- If the soil is coarse sand or gravel, none of the liquid methods works well, unless the hole is very large and a polyethylene sheet is used to contain the water or oil.

Nondestructive Methods

Nuclear density meter

(a) Direct transmission

(b) Backscatter

(c) Air gap

Principles

Density

The Gamma radiation is scattered by the soil particles and the amount of scatter is proportional to the total density of the material. The Gamma radiation is typically provided by the radium or a radioactive isotope of cesium.

Water content

The water content can be determined based on the neutron scatter by hydrogen atoms. Typical neutron sources are americium-beryllium isotopes.

Calibration

Calibration against compacted materials of known density is necessary, and for instruments operating on the surface the presence of an uncontrolled air gap can significantly affect the measurements.

SRI VIDYA COLLEGE OF ENGINEERING & TECHNOLOGY

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DEPARTMENT OF CIVIL ENGINEERING

CE6405-MECHANICS OF SOILS

UNIT-II

SOIL WATER & WATER FLOW



BY

Mr.R.PANDIARAJAN/AP/CIVIL

UNIT-II

SOIL WATER AND WATER FLOW

- * Soil is a three phase medium ----- solids, water, and air
- * Water in soils occur in various conditions
- * Water can flow through the voids in a soil from a point of high energy to a point of low energy.
- * Why studying flow of water in porous media.

1- To estimate the quantity of underground seepage

2- To determine the quantity of water that can be discharged from a soil

3- To determine the pore water pressure/effective geostatic stresses, and to analyze earth structures subjected to water flow.

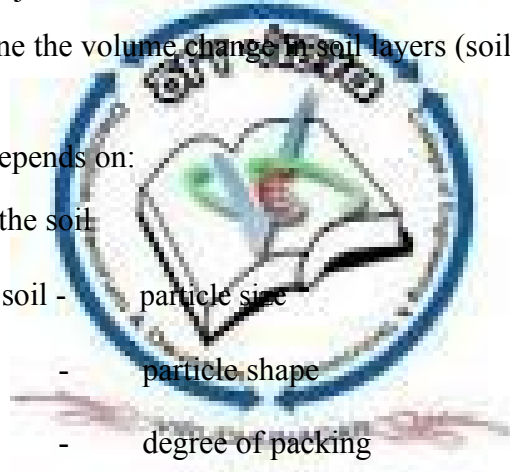
4- To determine the volume change in soil layers (soil consolidation) and settlement of foundation.

* Flow of Water in Soils depends on:

1- Porosity of the soil

2- Type of the soil -

- particle size
- particle shape
- degree of packing



3- Viscosity of the fluid - Temperature

- Chemical Components

4- Total head (difference in energy) - Pressure head

- Velocity head

- Elevation head

The degree of compressibility of a soil is expressed by the coefficient of permeability of the soil "k."

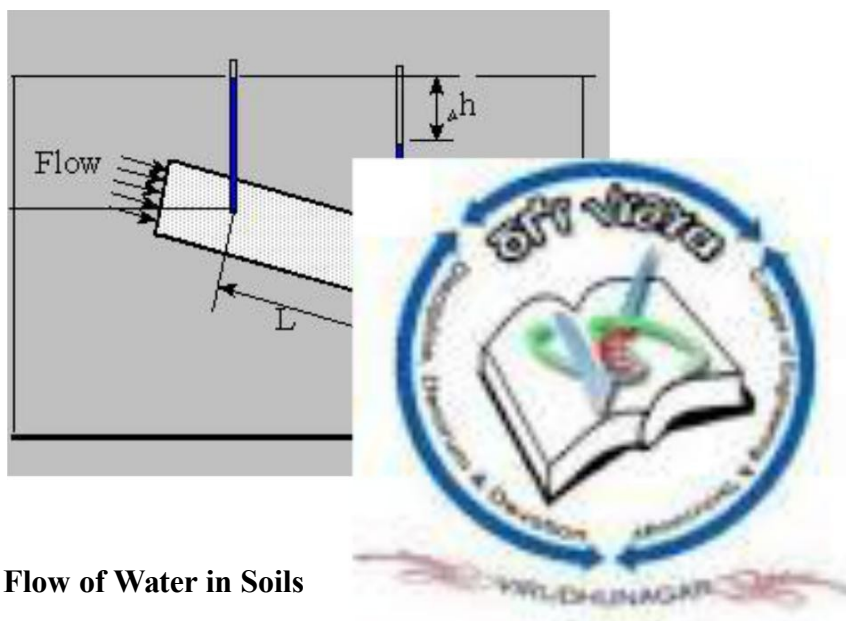
k cm/sec, ft/sec, m/sec,

Hydraulic Gradient Bernoulli's Equation:

$$h = Z + \frac{P}{\gamma_w} + \frac{v^2}{2g}$$

For soils

$$h = Z + \frac{P}{\gamma_w} + \frac{v^2}{2g}$$



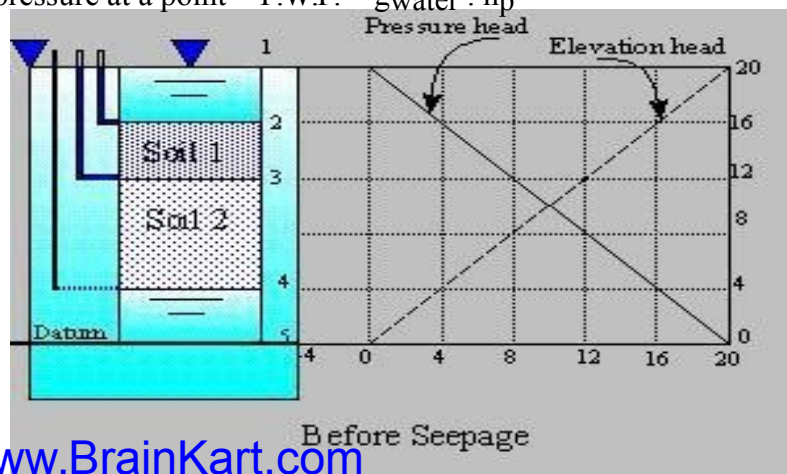
Flow of Water in Soils

1- Hydraulic Head in Soil

Total Head = Pressure head + Elevation Head

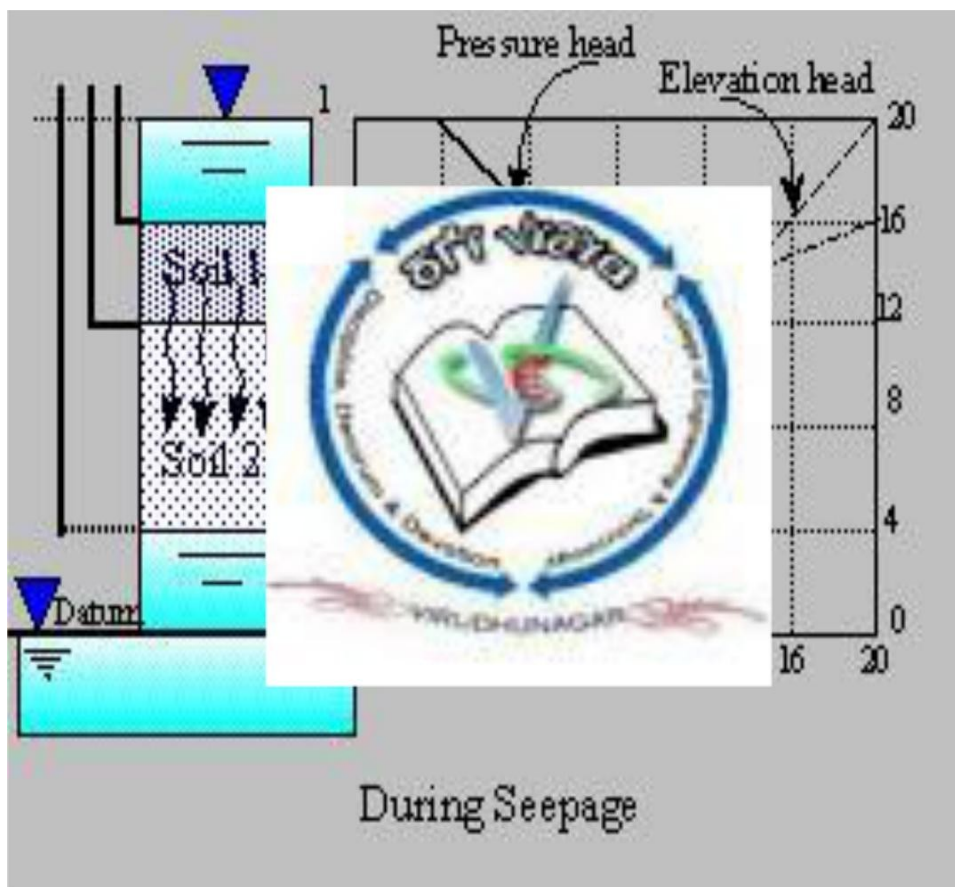
$$h_t = h_p + h_e$$

- Elevation head at a point = Extent of that point from the datum
- Pressure head at a point = Height of which the water rises in the piezometer above the point.
- Pore Water pressure at a point = P.W.P. = $\gamma_{\text{water}} \cdot h_p$



***How to measure the Pressure Head or the Piezometric Head?**

- 1- Assume that you do not have seepage in the system (Before Seepage)
- 2- Assume that you have piezometer at the point under consideration
- 3- Get the measurement of the piezometric head (Water column in the Piezometer before seepage) = $h_p(\text{Before Seepage})$
- 4- Now consider the problem during seepage
- 5- Measure the amount of the head loss in the piezometer (D_h) or the drop in the piezometric head.
- 6- The piezometric head during seepage = $h_p(\text{during seepage}) = h_p(\text{Before Seepage}) - D_h$



Formation of Clay Minerals

A soil particle may be a mineral or a rock fragment. A mineral is a chemical compound formed in nature during a geological process, whereas a rock fragment has a combination of one or more minerals. Based on the nature of atoms, minerals are classified as silicates, aluminates, oxides, carbonates and phosphates.

Out of these, silicate minerals are the most important as they influence the properties of clay soils. Different arrangements of atoms in the silicate minerals give rise to different silicate structures.

Basic Structural Units

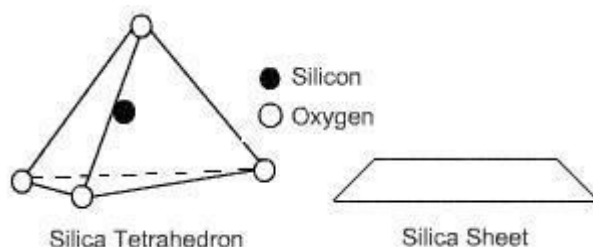
Soil minerals are formed from two basic structural units: tetrahedral and octahedral. Considering the valencies of the atoms forming the units, it is clear that the units are not electrically neutral and as such do not exist as single units.

The basic units combine to form sheets in which the oxygen or hydroxyl ions are shared among adjacent units. Three types of sheets are thus formed, namely silica sheet, gibbsite sheet and brucite sheet.

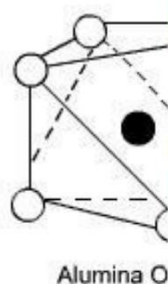
Isomorphous substitution is the replacement of the central atom of the tetrahedral or octahedral unit by another atom during the formation of the sheets.

The sheets then combine to form various two-layer or three-layer sheet minerals. As the basic units of clay minerals are sheet-like structures, the particle formed from stacking of the basic units is also plate-like. As a result, the surface area per unit mass becomes very large.

A tetrahedral unit consists of a central silicon atom that is surrounded by four oxygen atoms located at the corners of a tetrahedron. A combination of tetrahedrons forms a silica sheet.



An octahedral unit consists of a central ion, either aluminium or magnesium, that is surrounded by six hydroxyl ions located at the corners of an octahedron. A combination of aluminium-hydroxyl octahedrons forms a gibbsite sheet, whereas a combination of magnesium-hydroxyl octahedrons forms a brucite sheet.



Two-layer Sheet Minerals

Kaolinite and halloysite clay minerals are the most common.

Kaolinite Mineral

The basic kaolinite unit is a two-layer unit that is formed by stacking a gibbsite sheet on a silica sheet. These basic units are then stacked one on top of the other to form a lattice of the mineral. The units are held together by hydrogen bonds. The strong bonding does not permit water to enter the lattice. Thus, kaolinite minerals are stable and do not expand under saturation.

Kaolinite is the most abundant constituent of residual clay deposits.

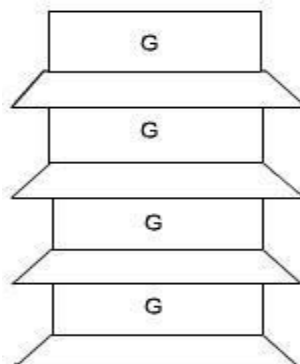


Fig: Kaolinite Mineral

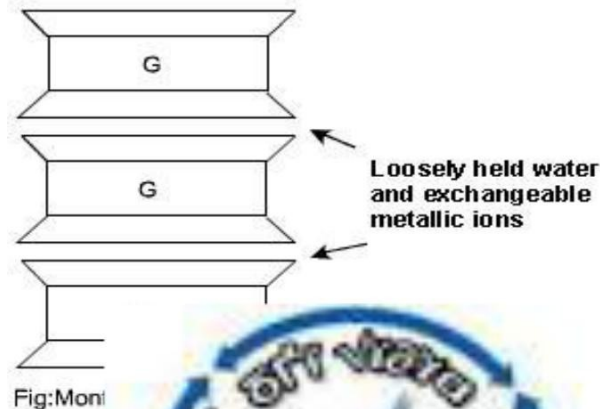
Halloysite Mineral

The basic unit is also a two-layer sheet similar to that of kaolinite except for the presence of wThree-layer Sheet Minerals

Montmorillonite and illite clay minerals are the most common. A basic three-layer sheet unit is formed by keeping one silica sheet each on the top and at the bottom of a gibbsite sheet. These units are stacked to form a lattice as shown.

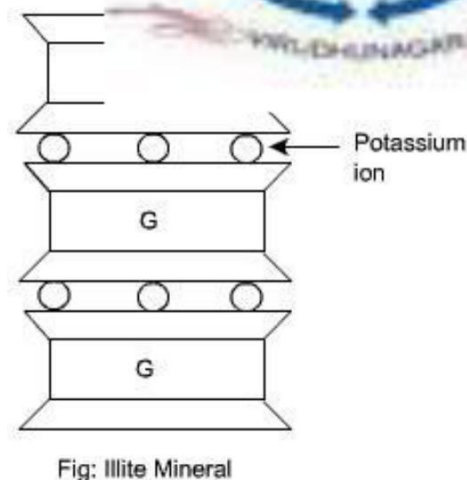
Montmorillonite Mineral

The bonding between the three-layer units is by van der Waals forces. This bonding is very weak and water can enter easily. Thus, this mineral can imbibe a large quantity of water causing swelling. During dry weather, there will be shrinkage.



Illite Mineral

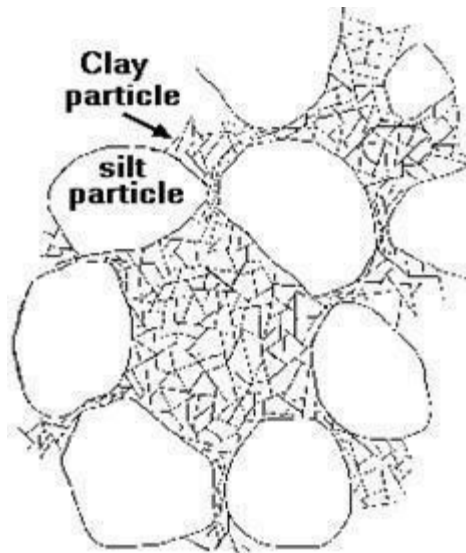
Illite consists of the basic montmorillonite units but are bonded by secondary valence forces and potassium ions, as shown. There is about 20% replacement of aluminium with silicon in the gibbsite sheet due to isomorphous substitution. This mineral is very stable and does not swell or shrink.



Fine Soil Fabric

Natural soils are rarely the same from one point in the ground to another. The content and nature of grains varies, but more importantly, so does the arrangement of these. The arrangement and organisation of particles and other features within a soil mass is termed its fabric.

CLAY particles are flaky. Their thickness is very small relative to their length & breadth, in some cases as thin as 1/100th of the length. They therefore have high specific surface values. These surfaces carry negative electrical charge, which attracts positive ions present in the pore water. Thus a lot of water may be held as adsorbed water within a clay mass.

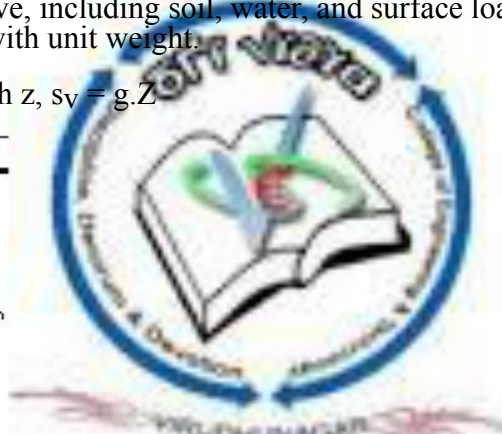
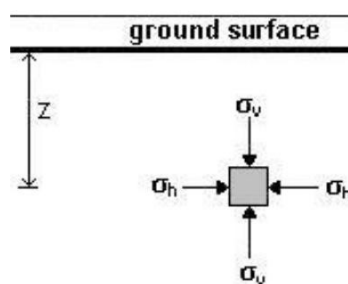


Stresses in the Ground

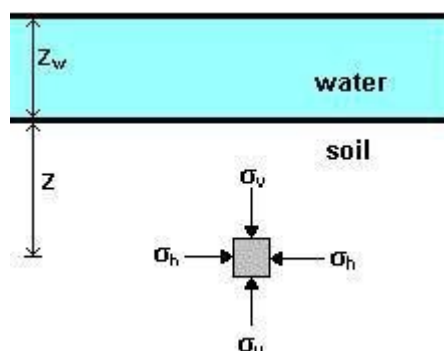
Total Stress

When a load is applied to soil, it is carried by the solid grains and the water in the pores. The total vertical stress acting at a point below the ground surface is due to the weight of everything that lies above, including soil, water, and surface loading. Total stress thus increases with depth and with unit weight.

Vertical total stress at depth z , $s_v = g \cdot Z$



Below a water body, the total stress is the sum of the weight of the soil up to the surface and the weight of water above this. $s_v = g \cdot Z + g_w \cdot Z_w$

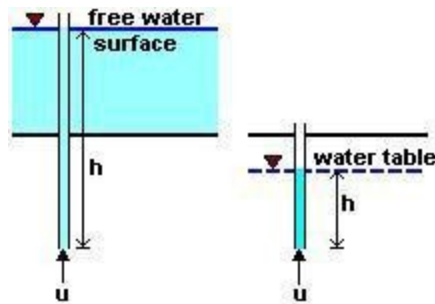


The total stress may also be denoted by s_z or just s . It varies with changes in water level and with excavation.

Pore Water Pressure

The pressure of water in the pores of the soil is called magnitude of pore water pressure depends on:

- the depth below the water table.
- the conditions of seepage flow



Under hydrostatic conditions, no water flow takes place, and the pore pressure at a given point is given by

$$u = \gamma_w \cdot h$$

where h = depth below water table or overlying water surface

It is convenient to think of pore water pressure as the pressure exerted by a column of water in an imaginary standpipe inserted at the given point.

The natural level of ground water is called the water table or the phreatic surface. Under conditions of no seepage flow, the water table is horizontal. The magnitude of the pore water pressure at the water table is zero. Below the water table, pore water pressures are positive.

Principle of Effective Stress

The principle of effective stress was enunciated by Karl Terzaghi in the year 1936. This principle is valid only for saturated soils, and consists of two parts:

1. At any point in a soil mass, the effective stress (represented by σ') is related to total stress (σ) and pore water pressure (u) as

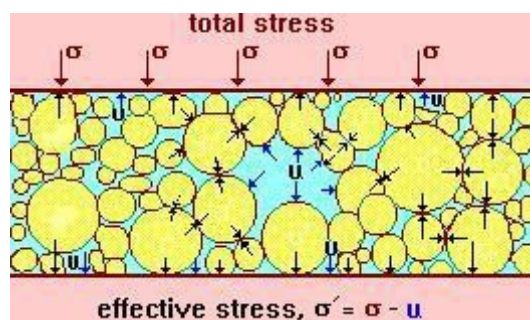
$$\sigma' = \sigma - u$$

Both the total stress and pore water pressure can be measured at any point.

2. All measurable effects of a change of stress, such as compression and a change of shearing resistance, are exclusively due to changes in effective stress.

$$\text{Compression} = f_1(\sigma')$$

$$\text{Shear Strength} = f_2(\sigma')$$



In a saturated soil system, as the voids are completely filled with water, the pore water pressure acts equally in all directions.

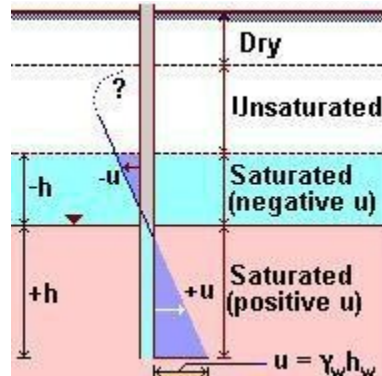
The effective stress is not the exact contact stress between particles but the distribution of load carried by the soil particles over the area considered. It cannot be measured and can only be computed.

If the total stress is increased due to additional load applied to the soil, the pore water pressure initially increases to counteract the additional stress. This increase in pressure

within the pores might cause water to drain out of the soil mass, and the load is transferred to the solid grains. This will lead to the increase of effective stress.

Effective Stress in Unsaturated Zone

Above the water table, when the soil is saturated, pore pressure will be negative (less than atmospheric). The height above the water table to which the soil is saturated is called the capillary rise, and this depends on the grain size and the size of pores. In coarse soils, the capillary rise is very small.



Between the top of the saturated zone and the ground surface, the soil is partially saturated, with a consequent reduction in unit weight. The pore pressure in a partially saturated soil consists of two components:

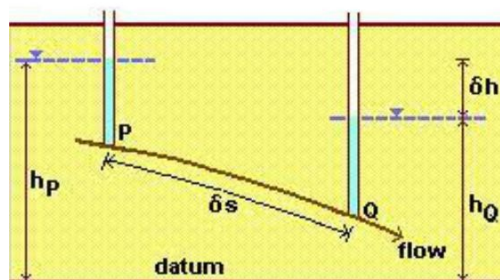
Pore water pressure = u_w

Pore air pressure = u_a

Water is incompressible, whereas air is compressible. The combined effect is a complex relationship involving partial pressures and the degree of saturation of the soil.

Effective Stress Under Hydrodynamic Conditions

There is a change in pore water pressure in conditions of seepage flow within the ground. Consider seepage occurring between two points P and Q. The potential driving the water flow is the hydraulic gradient between the two points, which is equal to the head drop per unit length. In steady state seepage, the gradient remains constant.

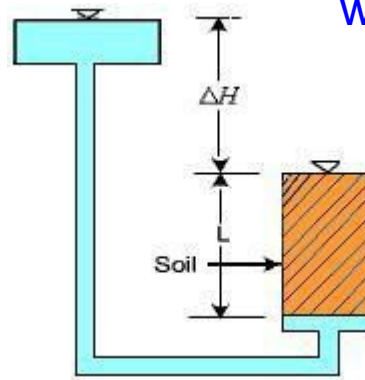


Hydraulic gradient from P to Q, $i = \delta h / \delta s$

As water percolates through soil, it exerts a drag on soil particles it comes in contact with. Depending on the flow direction, either downward or upward, the drag either increases or decreases inter-particle contact forces.

A downward flow increases effective stress.

In contrast, an upward flow opposes the force of gravity and can even cause to counteract completely the contact forces. In such a situation, effective stress is reduced to zero and the soil behaves like a very viscous liquid. Such a state is known as quick sand condition. In nature, this condition is usually observed in coarse silt or fine sand subject to artesian conditions.



At the bottom of the soil column,

$$\sigma = \gamma \cdot L$$

$$u = \gamma_w (L + \Delta H)$$

During quick sand condition, the effective stress is reduced to zero.

$$\gamma \cdot L = \gamma_w (L + \Delta H)$$

$$L(\gamma - \gamma_w) = \gamma_w \cdot \Delta H$$

$$L \cdot \gamma_s = \gamma_w \cdot \Delta H$$

$$\frac{\Delta H}{L} = \frac{\gamma_s}{\gamma_w} = i_{cr} \approx 1$$

where i_{cr} = critical hydraulic gradient

This shows that when water flows upward under a hydraulic gradient of about 1, it completely neutralizes the force on account of the weight of particles, and thus leaves the particles suspended in water.

The Importance of Effective Stress

At any point within the soil mass, the magnitudes of both total stress and pore water pressure are dependent on the ground water position. With a shift in the water table due to seasonal fluctuations, there is a resulting change in the distribution in pore water pressure with depth.

Changes in water level below ground result in changes in effective stresses below the water table. A rise increases the pore water pressure at all elevations thus causing a decrease in effective stress. In contrast, a fall in the water table produces an increase in the effective stress.

Changes in water level above ground do not cause changes in effective stresses in the ground below. A rise above ground surface increases both the total stress and the pore water pressure by the same amount, and consequently effective stress is not altered.

In some analyses it is better to work with the changes of quantity, rather than in absolute quantities. The effective stress expression then becomes:

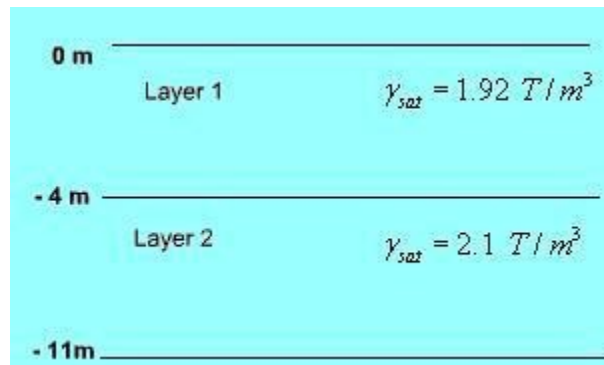
$$Ds' = Ds - Du$$

If both total stress and pore water pressure change by the same amount, the effective stress remain constant.

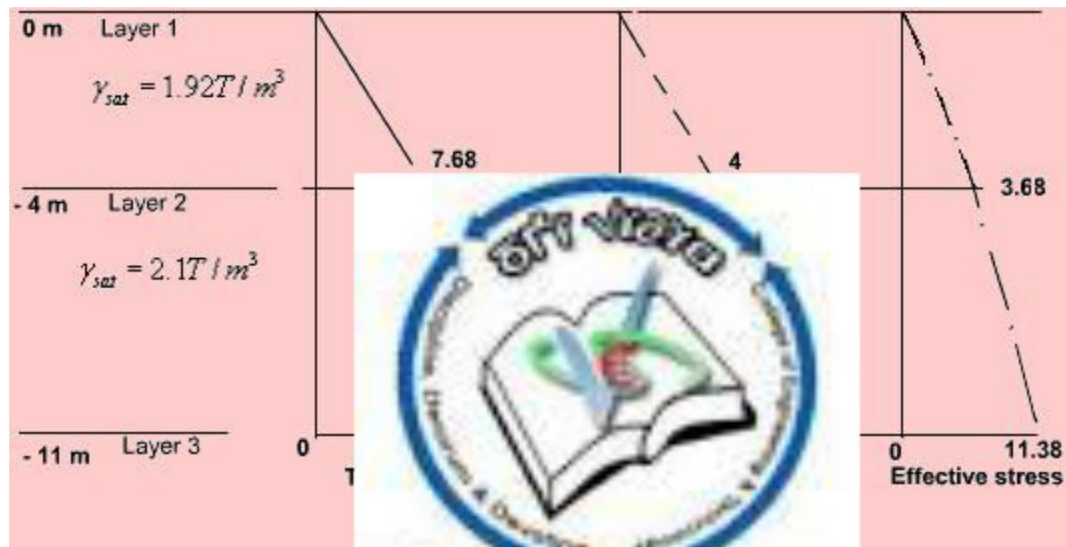
Total and effective stresses must be distinguishable in all calculations. Ground movements and instabilities can be caused by changes in total stress, such as caused by loading by foundations and unloading due to excavations. They can also be caused by changes in pore water pressures, such as failure of slopes after rainfall.

Worked Examples

Example 1: For the soil deposit shown below, draw the total stress, pore water pressure and effective stress diagrams. The water table is at ground level.



Solution:



Total stress

At -4m, $\sigma = 1.92 \times 4 = 7.68$

At -11m, $\sigma = 7.68 + 2.1 \times 7 = 22.38 \text{ T/m}^2$

Pore water pressure

At -4 m, $u = 1 \times 4 = 4 \text{ T/m}^2$

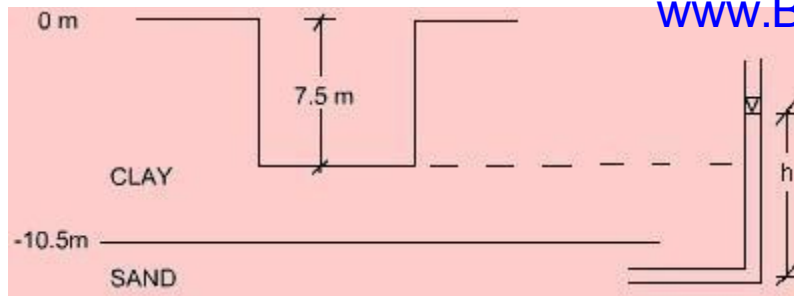
At -11 m, $u = 1 \times 11 = 11 \text{ T/m}^2$

Effective stress

At -4 m, $\sigma' = 7.68 - 4 = 3.68 \text{ T/m}^2$

At -11m, $\sigma' = 22.38 - 11 = 11.38 \text{ T/m}^2$

Example 2: An excavation was made in a clay stratum having $\gamma_t = 2 \text{ T/m}^3$. When the depth was 7.5 m, the bottom of the excavation cracked and the pit was filled by a mixture of sand and water. The thickness of the clay layer was 10.5 m, and below it was a layer of pervious water-bearing sand. How much was the artesian pressure in the sand layer?



Solution:

When the depth of excavation was 7.5 m, at the interface of the CLAY and SAND layers, the effective stress was equal to zero.

Downward pressure due to weight of clay = Upward pressure due to artesian pressure

$(10.5 - 7.5) \gamma_t = \gamma_w h$, where h = artesian

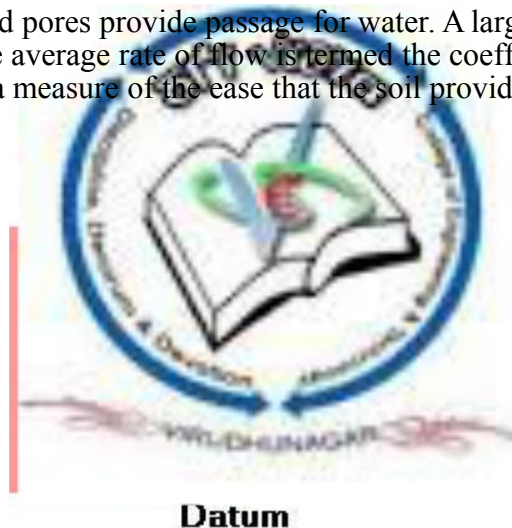
pressure head $3 \times 2 = 1 \times h$

$\therefore h = 6 \text{ m} = 0.6 \text{ kg/cm}^2$ or 6 T/m^2 artesian pressure

Permeability of Soils

Pressure, Elevation and Total Heads

In soils, the interconnected pores provide passage for water. A large number of such flow paths act together, and the average rate of flow is termed the coefficient of permeability, or just permeability. It is a measure of the ease that the soil provides to the flow of water through its pores.



At point A, the pore water pressure (u) can be measured from the height of water in a standpipe located at that point.

The height of the water column is the pressure head (h_w).

$$h_w = u / \gamma_w$$

To identify any difference in pore water pressure at different points, it is necessary to eliminate the effect of the points of measurement. With this in view, a datum is required from which locations are measured.

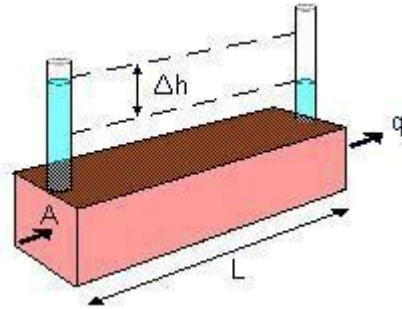
The elevation head (h_z) of any point is its height above the datum line. The height of water level in the standpipe above the datum is the piezometric head (h).

$$h = h_z + h_w$$

Total head consists of three components: elevation head, pressure head, and velocity head. As seepage velocity in soils is normally low, velocity head is ignored, and total head becomes equal to the piezometric head. Due to the low seepage velocity and small size of pores, the flow of water in the pores is steady and laminar in most cases. Water flow takes place between two points in soil due to the difference in total heads.

Darcy's Law

Darcy's law states that there is a linear relationship between flow velocity (v) and hydraulic gradient (i) for any given saturated soil under steady laminar flow conditions.



If the rate of flow is q (volume/time) through cross-sectional area (A) of the soil mass, Darcy's Law can be expressed as

$$v = q/A = k \cdot i$$

where k = permeability of the soil

$$i = Dh/L$$

Dh = difference in total heads

L = length of the soil mass

The flow velocity (v) is also called the Darcian velocity or the superficial velocity. It is different from the actual velocity inside the soil pores, which is known as the seepage

velocity, v_s . At the particulate level, the water follows a tortuous path through the pores. Seepage velocity is always greater than the superficial velocity, and it is expressed as:

$$v_s = \frac{v}{n}$$

where Av = Area of voids on a cross section normal to the direction of flow
 n = porosity of the soil

Permeability of Different Soils

Permeability (k) is an engineering property of soils and is a function of the soil type. Its value depends on the average size of the pores and is related to the distribution of particle sizes, particle shape and soil structure. The ratio of permeabilities of typical sands/gravels to

those of typical clays is of the order of 10^6 . A small proportion of fine material in a coarse-grained soil can lead to a significant reduction in permeability.

For different soil types as per grain size, the orders of magnitude for permeability are as follows:

Soil	k (cm/sec)
Gravel	10
Coarse sand	10 to 10
Medium sand	10 to 10
Fine sand	10 to 10
Silty sand	10 to 10
Silt	1×10
Clay	10^{-7} to 10^{-9}

Factors affecting Permeability

In soils, the permeant or pore fluid is mostly water whose variation in property is generally very less. Permeability of all soils is strongly influenced by the density of packing of the soil particles, which can be represented by void ratio (e) or porosity (n).

For Sands

In sands, permeability can be empirically related to the square of some representative grain size from its grain-size distribution. For filter sands, Allen Hazen in 1911 found that $k \propto (D_{10})^2$ cm/s where D_{10} = effective grain size in cm.

Different relationships have been attempted relating void ratio and permeability, such as $k \propto \frac{e^3}{(1+e)}$, and $k \propto \frac{e^3}{1+e} \cdot \frac{\gamma_w}{\eta}$. They have been obtained from the Kozeny-Carman equation for laminar flow in saturated soils.

$$k = \frac{1}{k_0 k_T S_s^2} \cdot \frac{e^3}{1+e} \cdot \frac{\gamma_w}{\eta}$$

where k_0 and k_T are factors depending on the shape and tortuosity of the pores respectively, S_s

is the surface area of the solid particles per unit volume of solid material, and γ_w and η are unit weight and viscosity of the pore water. The equation can be reduced to a simpler form as

$$k = C \cdot \frac{e^3}{1+e} \approx C \cdot e^2$$

For Silts and Clays

For silts and clays, the Kozeny-Carman equation does not work well, and $\log k$ versus e plot has been found to indicate a linear relationship.

For clays, it is typically found that

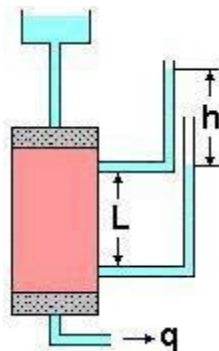
$$\log_{10} k = C_k (e - e_k)$$

where C_k is the permeability change index and e_k is a reference void ratio.

Laboratory Measurement of Permeability

Constant Head Flow

Constant head permeameter is recommended for coarse-grained soils only since for such soils, flow rate is measurable with adequate precision. As water flows through a sample of cross-section area A , steady total head drop h is measured across length L .

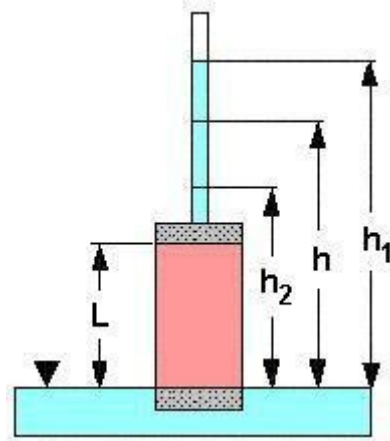


Permeability k is obtained from:

$$k = \frac{qL}{Ah}$$

Falling Head Flow

Falling head permeameter is recommended for fine-grained soils.



Total head h in standpipe of area a is allowed to fall. Hydraulic gradient varies with time.

Heads h_1 and h_2 are measured at times t_1 and t_2 . At any time t , flow through the soil sample of cross-sectional area A is

$$q = k \cdot h \cdot \frac{A}{L} \quad \text{----- (1)}$$

Flow in unit time through the standpipe of cross-sectional area a is

$$= a \times \left(-\frac{dh}{dt} \right) \quad \text{(2)}$$

Equating (1) and (2),

$$\begin{aligned} -a \cdot \frac{dh}{dt} &= k \cdot h \cdot \frac{A}{L} \\ -\frac{dh}{h} &= \left(\frac{kA}{La} \right) dt \\ \text{or} \end{aligned}$$

Integrating between the limits,

$$\begin{aligned} \log_e \left(\frac{h_1}{h_2} \right) &= \frac{k \cdot A}{L \cdot a} (t_2 - t_1) \\ k &= \frac{L \cdot a \cdot \log_e \left(\frac{h_1}{h_2} \right)}{A(t_2 - t_1)} \\ &= \frac{2.3 L \cdot a \cdot \log_{10} \left(\frac{h_1}{h_2} \right)}{A(t_2 - t_1)} \end{aligned}$$

Field Tests for Permeability

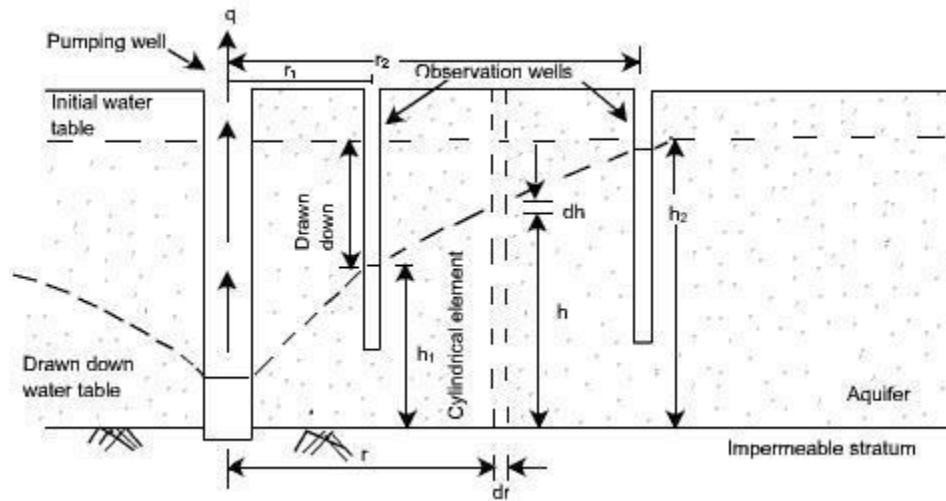
Field or in-situ measurement of permeability avoids the difficulties involved in obtaining and setting up undisturbed samples in a permeameter. It also provides information about bulk permeability, rather than merely the permeability of a small sample.

A field permeability test consists of pumping out water from a main well and observing the resulting drawdown surface of the original horizontal water table from at least two observation wells. When a steady state of flow is reached, the flow quantity and the levels

in the observation wells are noted.

Two important field tests for determining permeability are: Unconfined flow pumping test, and confined flow pumping test.

Unconfined Flow Pumping Test



In this test, the pumping causes a drawdown in an unconfined (i.e. open surface) soil stratum, and generates a radial flow of water towards the pumping well. The steady-state heads h_1 and h_2 in observation wells at radii r_1 and r_2 are monitored till the flow rate q becomes steady.

The rate of radial flow through any cylindrical surface around the pumping well is equal to the amount of water pumped out. Consider such a surface having radius r , thickness dr and height h . The hydraulic gradient is

$$i = \frac{dh}{dr}$$

Area of flow, $A = 2\pi rh$

From Darcy's Law,

$$q = k \cdot i \cdot A$$

$$= k \cdot \frac{dh}{dr} \cdot 2\pi rh$$

Arranging and integrating,

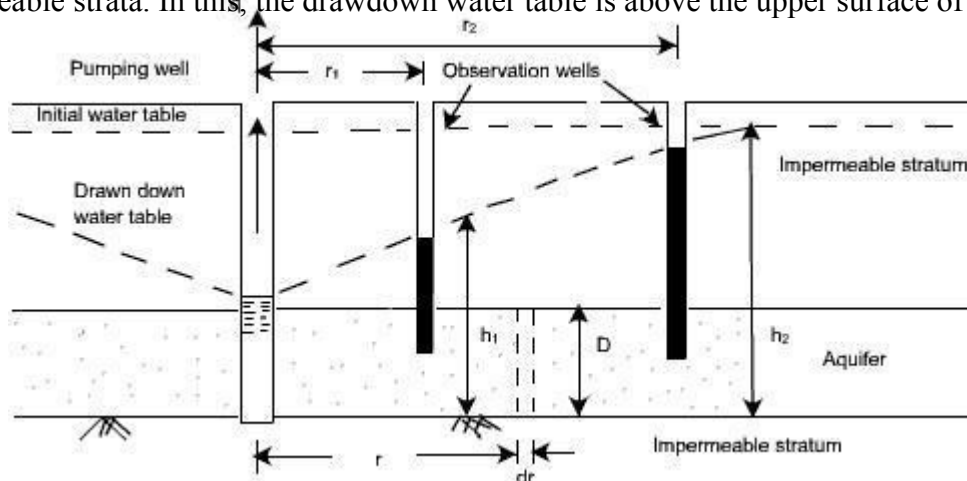
$$\int_{r_1}^{r_2} \frac{dr}{r} = \int_{h_1}^{h_2} \frac{2r}{q} \cdot K \cdot h \cdot dh$$

$$k = \frac{q \cdot \log_e \left(\frac{r_2}{r_1} \right)}{r(h_2^2 - h_1^2)}$$

Field Tests for Permeability

Confined Flow Pumping Test

Artesian conditions can exist in a aquifer of thickness D confined both above and below by impermeable strata. In this, the drawdown water table is above the upper surface of the aquifer.



For a cylindrical surface at radius r ,

$$q = k \cdot \frac{dh}{dr} \cdot 2\pi r D$$

$$\int_{r_1}^{r_2} \frac{dr}{r} = \int_{h_1}^{h_2} \frac{2\pi D k}{q}$$

Integrating,

$$\log_e \left(\frac{r_2}{r_1} \right) = \frac{2\pi D k}{q} (h_2 - h_1)$$

$$k = \frac{q \cdot \log_e \left(\frac{r_2}{r_1} \right)}{2\pi r D (h_2 - h_1)}$$



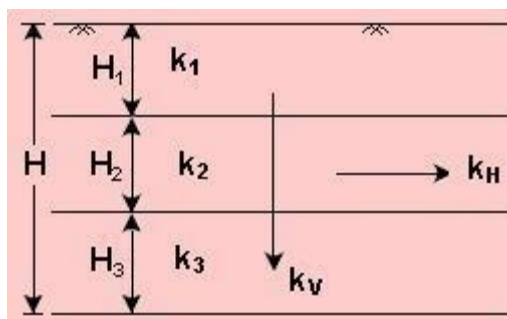
Permeability of Stratified Deposits

When a soil deposit consists of a number of horizontal layers having different permeabilities, the average value of permeability can be obtained separately for both vertical

flow and horizontal flow, as k_v and k_H respectively.

Consider a stratified soil having horizontal layers of thickness H_1, H_2, H_3 , etc.

with coefficients of permeability k_1, k_2, k_3 , etc.



For vertical flow

The flow rate q through each layer per unit area is the same.

$$q = q_1 = q_2 = \dots$$

Let i be the equivalent hydraulic gradient over the total thickness H and let the hydraulic gradients in the layers be i_1, i_2, i_3 , etc. respectively.

$k_v \cdot i = k_1 \cdot i_1 = k_2 \cdot i_2 = \dots$ where k_v = Average vertical permeability

$$k_v \cdot \frac{h}{H} = k_1 \cdot \frac{h_1}{H_1} = k_2 \cdot \frac{h_2}{H_2} = \dots$$

The total head drop h across the layers is

$$h = h_1 + h_2 + \dots$$

$$h = \frac{k_v \cdot h}{H} \cdot \frac{H_1}{k_1} + \frac{k_v \cdot h}{H} \cdot \frac{H_2}{k_2} + \dots$$

$$k_v = \frac{H}{\frac{H_1}{k_1} + \frac{H_2}{k_2} + \dots}$$

Horizontal flow

When the flow is horizontal, the hydraulic gradient is the same in each layer, but the quantity of flow is different in each layer.

$$i = i_1 = i_2 = i_3 = \dots$$

The total flow is

$$Q = Q_1 + Q_2 + Q_3 + \dots$$

Considering unit width normal to the cross-section plane,

$$k_H \cdot i \cdot H = k_1 \cdot i_1 \cdot H_1 + k_2 \cdot i_2 \cdot H_2 + \dots$$

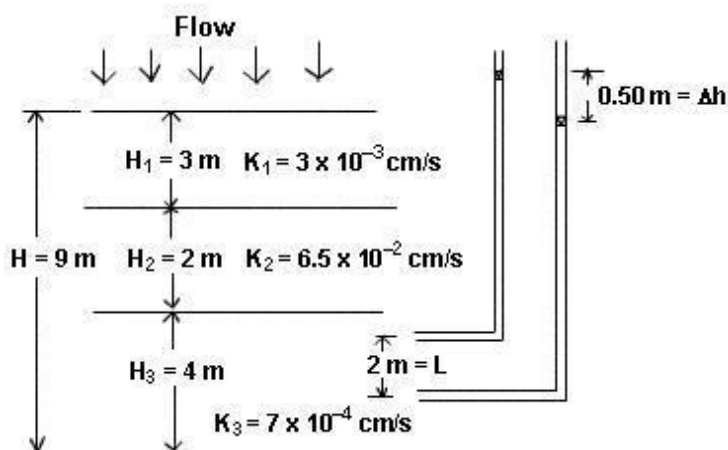
$$k_H = \frac{1}{H} (k_1 \cdot H_1 + k_2 \cdot H_2 + \dots)$$



Worked Examples

Example 1: Determine the following:

- Equivalent coefficient of vertical permeability of the three layers
- The rate of flow per m^2 of plan area
- The total head loss in the three layers



Solution:

$$K_v = \frac{H}{\frac{H_1}{K_1} + \frac{H_2}{K_2} + \frac{H_3}{K_3}} = \frac{9}{\frac{3}{3 \times 10^{-3}} + \frac{2}{6.5 \times 10^{-2}} + \frac{4}{7 \times 10^{-4}}} = 1.33 \times 10^{-3} \text{ cm/s}$$

(b) Considering an area $A = 1 \text{ m}^2 = 1 \times 10^4 \text{ cm}^2$

$$q = k \cdot i \cdot A = k_3 \cdot \frac{\Delta h}{L} \cdot A = 7 \times 10^{-4} \times 0.25/2 \times (1 \times 10^4) = 0.875 \text{ cm}^3/\text{s per m}^2 \text{ of plan area}$$

(c) For continuity of flow, velocity is the same.

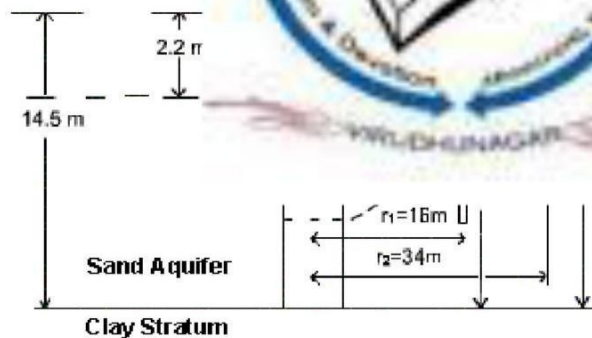
$$k_3 \cdot \frac{\Delta h}{L} = k_v \cdot \frac{\Delta h_{\text{total}}}{H}, \text{ where } \Delta h_{\text{total}} = \text{total head loss in three layers}$$

$$\therefore \Delta h_{\text{total}} = k_3 \cdot \frac{\Delta h}{L} \cdot \frac{H}{k_v} = 7 \times 10^{-4} \times \frac{0.50}{2} \times \frac{9}{1.33 \times 10^{-3}} = 1.184 \text{ m}$$

Example 2: For a field pumping test, a well was sunk through a horizontal stratum of sand 14.5 thick and underlain by a clay stratum. Two observation wells were sunk at horizontal distances of 16 m and 34 m respectively from the pumping well. The initial position of the water table was 2.2 m below ground level.

At a steady-state pumping rate of 1850 litres/min, the drawdowns in the observation wells were found to be 2.45 m and 1.20 m respectively. Calculate the coefficient of permeability of the sand.

Solution:



$$k = \frac{q \cdot \log_e \left(\frac{r_2}{r_1} \right)}{r (h_2^2 - h_1^2)}$$

$$q = 1850 \text{ litres/min} = \frac{1850 \times 10^{-3}}{60} \text{ m}^3/\text{s}$$

$$r_1 = 16 \text{ m}$$

$$r_2 = 34 \text{ m}$$

$$h_1 = 14.5 - 2.2 - 2.45 = 9.85 \text{ m}$$

$$h_2 = 14.5 - 2.2 - 1.2 = 11.1 \text{ m}$$

$$\frac{1850 \times 10^{-3}}{60} \times \log_e \left(\frac{34}{16} \right)$$

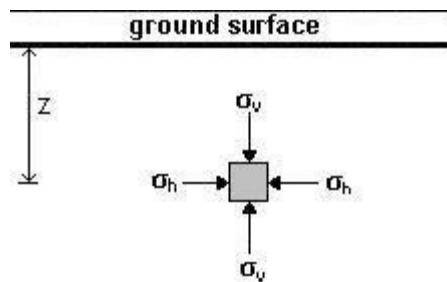
$$= 2.82 \times 10^{-4} \text{ m/s} = 1.41 \times 10^{-2} \text{ cm/s}$$

Stresses in the Ground

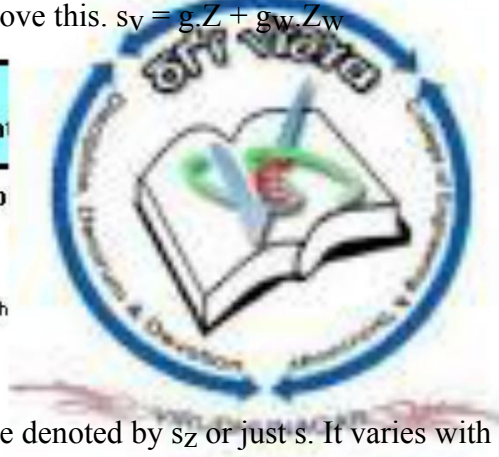
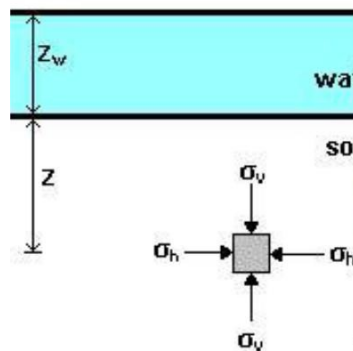
Total Stress

When a load is applied to soil, it is carried by the solid grains and the water in the pores. The total vertical stress acting at a point below the ground surface is due to the weight of everything that lies above, including soil, water, and surface loading. Total stress thus increases with depth and with unit weight.

Vertical total stress at depth z , $s_v = g \cdot Z$



Below a water body, the total stress is the sum of the weight of the soil up to the surface and the weight of water above this. $s_v = g \cdot Z + g_w \cdot Z_w$

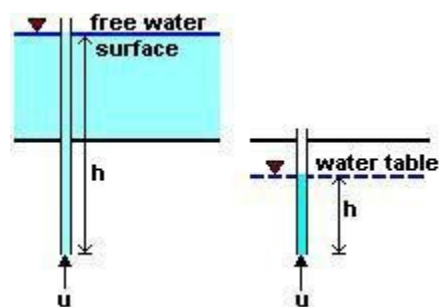


The total stress may also be denoted by s_z or just s . It varies with changes in water level and with excavation.

Pore Water Pressure

The pressure of water in the pores of the soil is called pore water pressure (u). The magnitude of pore water pressure depends on:

- the depth below the water table.
- the conditions of seepage flow.



Under hydrostatic conditions, no water flow takes place, and the pore pressure at a given point is given by

$$u = g_w \cdot h$$

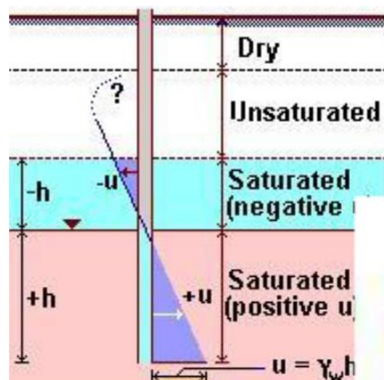
where h = depth below water table or overlying water surface

It is convenient to think of pore water pressure as the pressure exerted by a column of water in an imaginary standpipe inserted at the given point.

The natural level of ground water is called the water table or the phreatic surface. Under conditions of no seepage flow, the water table is horizontal. The magnitude of the pore water pressure at the water table is zero. Below the water table, pore water pressures are positive.

Effective Stress in Unsaturated Zone

Above the water table, when the soil is saturated, pore pressure will be negative (less than atmospheric). The height above the water table to which the soil is saturated is called the capillary rise, and this depends on the grain size and the size of pores. In coarse soils, the capillary rise is very small.



Between the top of the saturated zone and the ground surface, the soil is partially saturated, with a consequent reduction in unit weight. The pore pressure in a partially saturated soil consists of two components:

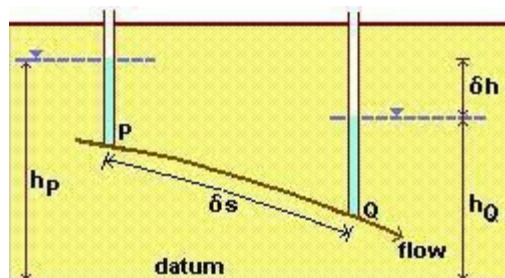
Pore water pressure = u_w

Pore air pressure = u_a

Water is incompressible, whereas air is compressible. The combined effect is a complex relationship involving partial pressures and the degree of saturation of the soil.

Effective Stress under Hydrodynamic Conditions

There is a change in pore water pressure in conditions of seepage flow within the ground. Consider seepage occurring between two points P and Q. The potential driving the water flow is the hydraulic gradient between the two points, which is equal to the head drop per unit length. In steady state seepage, the gradient remains constant.

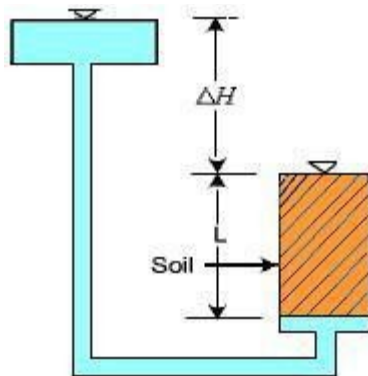


Hydraulic gradient from P to Q, $i = dh/ds$

As water percolates through soil, it exerts a drag on soil particles it comes in contact with. Depending on the flow direction, either downward or upward, the drag either increases or decreases inter-particle contact forces.

A downward flow increases effective stress.

In contrast, an upward flow opposes the force of gravity and can even cause to counteract completely the contact forces. In such a situation, effective stress is reduced to zero and the soil behaves like a very viscous liquid. Such a state is known as quick sand condition. In nature, this condition is usually observed in coarse silt or fine sand subject to artesian conditions.



At the bottom of the soil column,

$$\sigma = \gamma_s L$$

$$u = \gamma_w (L + \Delta H)$$

During quick sand condition, the effective stress is reduced to zero.

$$\gamma_s L = \gamma_w (L + \Delta H)$$

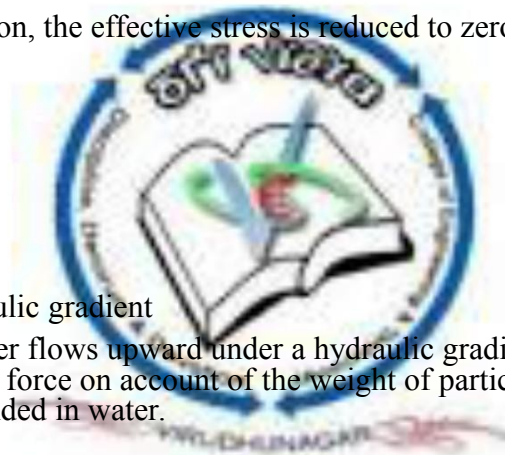
$$L(\gamma_s - \gamma_w) = \gamma_w \Delta H$$

$$L \gamma_d = \gamma_w \Delta H$$

$$\frac{\Delta H}{L} = \frac{\gamma_d}{\gamma_w} = i_{cr} \approx 1$$

Where i_{cr} = critical hydraulic gradient

This shows that when water flows upward under a hydraulic gradient of about 1, it completely neutralizes the force on account of the weight of particles, and thus leaves the particles suspended in water.



The Importance of Effective Stress

At any point within the soil mass, the magnitudes of both total stress and pore water pressure are dependent on the ground water position. With a shift in the water table due to seasonal fluctuations, there is a resulting change in the distribution in pore water pressure with depth. Changes in water level below ground result in changes in effective stresses below the water table. A rise increases the pore water pressure at all elevations thus causing a decrease in effective stress. In contrast, a fall in the water table produces an increase in the effective stress.

Changes in water level above ground do not cause changes in effective stresses in the ground below. A rise above ground surface increases both the total stress and the pore water pressure by the same amount, and consequently effective stress is not altered.

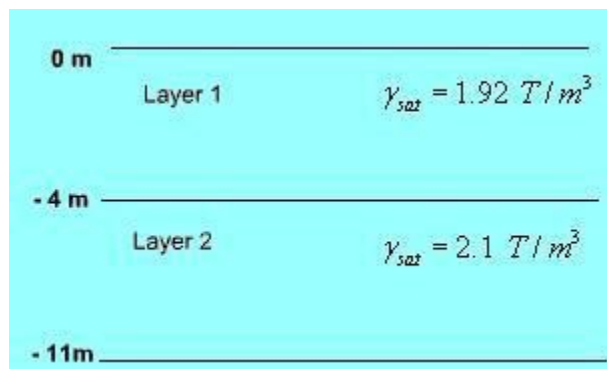
In some analyses it is better to work with the changes of quantity, rather than in absolute quantities. The effective stress expression then becomes:

$$D_s' = D_s - D_u$$

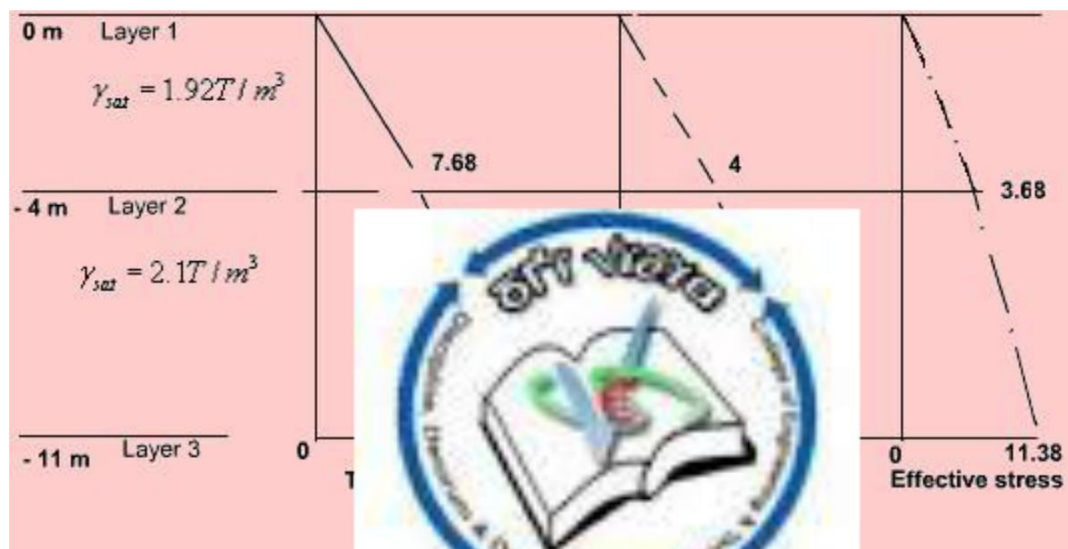
If both total stress and pore water pressure change by the same amount, the effective stress remains constant.

Total and effective stresses must be distinguishable in all calculations. Ground movements and instabilities can be caused by changes in total stress, such as caused by loading by foundations and unloading due to excavations. They can also be caused by changes in pore water pressures, such as failure of slopes after rainfall.

Example 1: For the soil deposit shown below, draw the total stress, pore water pressure and effective stress diagrams. The water table is at ground level.



Solution:



Total stress

At - 4 m, $\sigma = 1.92 \times 4 = 7.68$

At -11 m, $\sigma = 7.68 + 2.1 \times 7 = 22.38 \text{ T/m}^2$

Pore water pressure

At - 4 m, $u = 1 \times 4 = 4 \text{ T/m}^2$

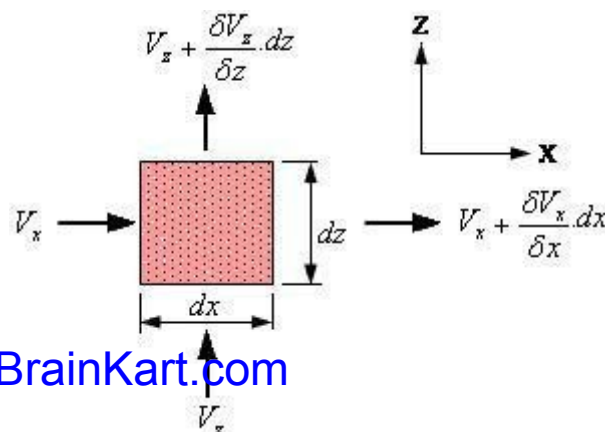
At -11 m, $u = 1 \times 11 = 11 \text{ T/m}^2$

Effective stress

At - 4 m, $\bar{\sigma} = 7.68 - 4 = 3.68 \text{ T/m}^2$

At -11 m, $\bar{\sigma} = 22.38 - 11 = 11.38 \text{ T/m}^2$

Seepage in Soils



A rectangular soil element is shown with dimensions dx and dz in the plane, and thickness dy perpendicular to this plane. Consider planar flow into the rectangular soil element.

In the x -direction, the net amount of the water entering and leaving the element is

$$\frac{\partial V_x}{\partial x} dx dy dz$$

Similarly in the z -direction, the difference between the water inflow and outflow is

$$\frac{\partial V_z}{\partial z} dz dx dy$$

For a two-dimensional steady flow of pore water, any imbalance in flows into and out of an element in the z -direction must be compensated by a corresponding opposite imbalance in the x -direction. Combining the above, and dividing by $dx dy dz$, the continuity equation is expressed as

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_z}{\partial z} = 0$$

From Darcy's law, $V_x = k_x \cdot \frac{\partial h}{\partial x}$, where h is the head causing flow.

When the continuity equation is combined with Darcy's law, the equation for flow is expressed as:

$$k_x \cdot \frac{\partial^2 h}{\partial x^2} + k_z \cdot \frac{\partial^2 h}{\partial z^2} = 0$$

For an isotropic material in which the permeability is the same in all directions (i.e. $k_x = k_z$), the flow equation is

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0$$

This is the Laplace equation governing two-dimensional steady state flow. It can be solved graphically, analytically, numerically, or analogically.

For the more general situation involving three-dimensional steady flow, Laplace equation

becomes:
$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0$$

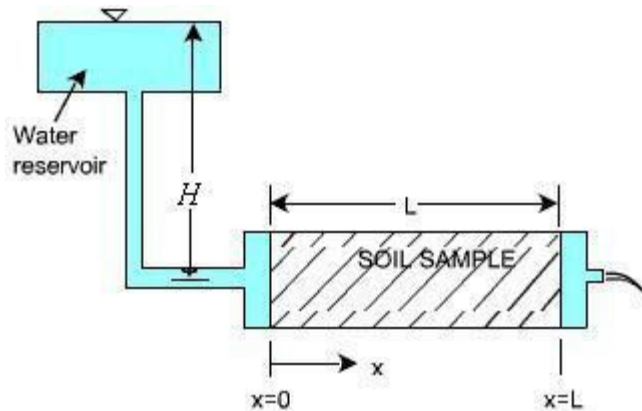
One-dimensional Flow

For this, the Laplace Equation is
$$\frac{\partial^2 h}{\partial x^2} = 0$$

Integrating twice, a general solution is obtained.

$$\frac{\partial h}{\partial x} = c_1$$

The values of constants can be determined from the specific boundary conditions.



As shown, at $x = 0$, $h = H$, and at $x = L$, $h = 0$

Substituting and solving,

$$c_2 = H, \quad c_1 = -\frac{H}{L}$$

The specific solution for flow in the above permeameter is

$$h = H - \frac{H}{L}x$$

which states that head is dissipated in a linearly uniform manner over the entire length of the permeameter.



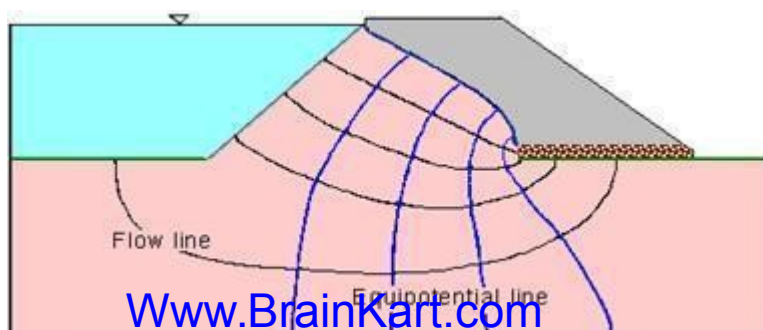
Two-dimensional Flow

Flow Nets

Graphical form of solutions to Laplace equation for two-dimensional seepage can be presented as flow nets. Two orthogonal sets of curves form a flow net:

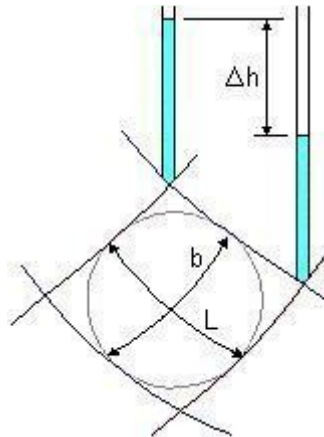
- Equipotential lines connecting points of equal total head h
- Flow lines indicating the direction of seepage down a hydraulic gradient

Two flow lines can never meet and similarly, two equipotential lines can never meet. The space between two adjacent flow lines is known as a flow channel, and the figure formed on the flownet between any two adjacent flow lines and two adjacent equipotential lines is referred to as a field. Seepage through an embankment dam is shown.



Calculation of flow in a channel

If standpipe piezometers were inserted into the ground with their tips on a single equipotential line, then the water would rise to the same level in each standpipe. The pore pressures would be different because of their different elevations. There can be no flow along an equipotential line as there is no hydraulic gradient.



Consider a field of length L within a flow channel. There is a fall of total head Dh . The average hydraulic gradient is

$$i = \frac{\Delta h}{L}$$

As the flow lines are b apart and considering unit length perpendicular to field, the flow rate is

$$\Delta q = kb \frac{\Delta h}{L}$$

There is an advantage in sketching flow nets in the form of curvilinear 'squares' so that a circle can be inscribed within each four-sided figure bounded by two equipotential lines and two flow lines.



In such a square, $b = L$, and the flow rate is obtained as $Dq = k.Dh$

Thus the flow rate through such a flow channel is the permeability k multiplied by the uniform interval Dh between adjacent equipotential lines.

Calculation of total flow

For a complete problem, the flow net can be drawn with the overall head drop h divided into N_d so that $Dh = h / N_d$.

If N_f is the no. of flow channels, then the total flow rate is

$$q = \Delta q.N_f = k.h \cdot \frac{N_f}{N_d}$$

Procedure for Drawing Flow Nets

At every point (x,z) where there is flow, there will be a value of head $h(x,z)$. In order to represent these values, contours of equal head are drawn.

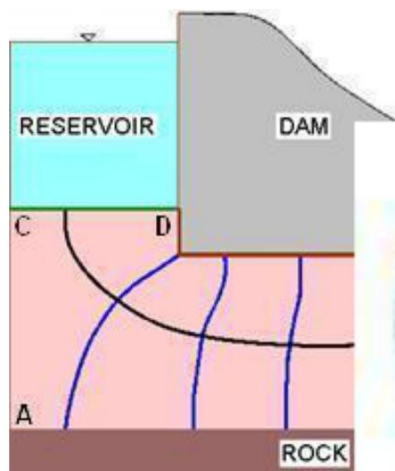
A flow net is to be drawn by trial and error. For a given set of boundary conditions, the flow net will remain the same even if the direction of flow is reversed. Flow nets are constructed such that the head lost between successive equipotential lines is the same, say Dh . It is useful in visualising the flow in a soil to plot the flow lines, as these are lines that are tangential to the flow at any given point. The steps of construction are:

1. Mark all boundary conditions, and draw the flow cross section to some convenient scale.
2. Draw a coarse net which is consistent with the boundary conditions and which has orthogonal equipotential and flow lines. As it is usually easier to visualise the pattern of flow, start by drawing the flow lines first.
3. Modify the mesh such that it meets the conditions outlined above and the fields between adjacent flow lines and equipotential lines are 'square'.
4. Refine the flow net by repeating step 3.

The most common boundary conditions are:

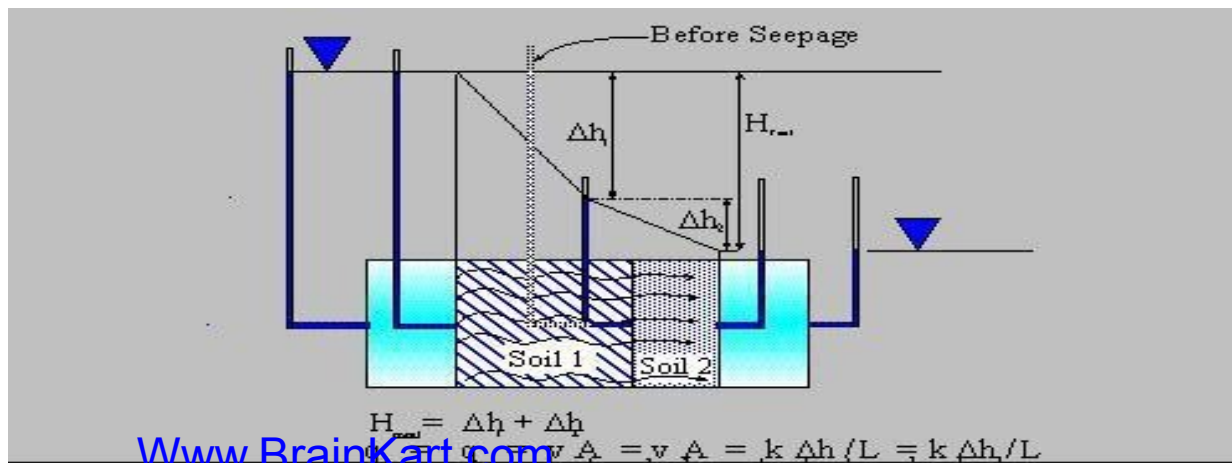
- A submerged permeable soil boundary is an equipotential line. This could have been determined by considering imaginary standpipes placed at the soil boundary, as for every point the water level in the standpipe would be the same as the water level. (Such a boundary is marked as CD and EF in the following figure.)
- The boundary between permeable and impermeable soil materials is a flow line (This is marked as AB in the same figure).
- Equipotential lines intersecting a phreatic surface do so at equal vertical intervals.

Uses of Flow Nets



The graphical properties of a flow net can be used in obtaining solutions for many seepage problems such as:

1. Estimation of seepage losses from reservoirs: It is possible to use the flow net in the transformed space to calculate the flow underneath the dam.
2. Determination of uplift pressures below dams: From the flow net, the pressure head at any point at the base of the dam can be determined. The uplift pressure distribution along the base can be drawn and then summed up.
3. Checking the possibility of piping beneath dams: At the toe of a dam when the upward exit hydraulic gradient approaches unity, boiling condition can occur leading to erosion in soil and consequent piping. Many dams on soil foundations have failed because of a sudden formation of a piped shaped discharge channel. As the stored water rushes out, the channel widens and catastrophic failure results. This is also often referred to as piping failure



SRI VIDYA COLLEGE OF ENGINEERING & TECHNOLOGY

VIRUDHUNAGAR

DEPARTMENT OF CIVIL ENGINEERING

CE6405-MECHANICS OF SOILS

UNIT-III

**. STRESS DISTRIBUTION, COMPRESSIBILITY AND
SETTLEMENT**



BY

Mr.R.PANDIARAJAN/AP/CIVIL

Terzaghi's Theory of one dimensional consolidation

The Theoretical concept of the consolidation process was developed by Terzaghi (1923)

Assumptions:

- (i) Soil is homogeneous & fully saturated
- (ii) Soil particles & water are incompressible
- (iii) The deformation of soil is due to change in volume
- (iv) Darcy's law for the velocity of flow of H_2O through soil is perfectly valid.
- (v) Coefficient of permeability is constant during consolidation.
- (vi) Load applied occurs only in one direction
- (vii) Coefficient of volume compressibility is constant during the process
- (viii) Constant value of initial water content



Terzaghi theory of consolidation of saturated clayey soils.

The differential equation for consolidation is

$$\frac{\partial u}{\partial t} = C_v \cdot \frac{\partial^2 u}{\partial z^2}$$

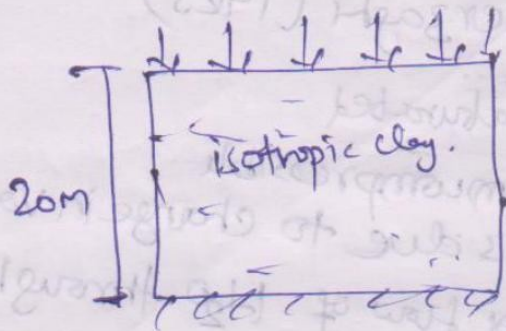
where C_v = coefficient of consolidation $C_v = \frac{k}{\gamma_w m_v}$

Problem:

A 20m thick isotropic clay stratum overlies an impervious rock. The coefficient of consolidation of soil is $5 \times 10^{-4} \text{ cm}^2/\text{sec}$ (C_v).

Find the time required for 50% & 90% consolidation. Time factor for $U=50\%$ is 0.85 where U is the degree of consolidation.

(ii) In order to accelerate the settlement rate, vertical drains of 1m diameter were made at 5m centre to centre in the soil stratum throughout the area.



$$t = \frac{T \cdot d^2}{C_v}$$

t = the time required time for the % . T = time factor

d = maximum

$$t_{90} = \frac{0.85 \times 5 \times 5}{5 \times 5}$$

$$t_{90} = \frac{0.85 \times 5 \times 5}{5 \times 5}$$

$$t_{90} = 215$$



$$t_{50} = \frac{0.2 \times 20 \times 100 \times 20 \times 100}{5 \times 10^4} = \frac{0.2 \times 20 \times 100 \times 20 \times 100 \times 4}{5 \times 10^3 \times 365 \times 24 \times 60 \times 60}$$

$$t_{50} = 50.74 \text{ years.}$$

A compressible layer is expected to have a total settlement of 15cm under a given loading. It settles by 3cm at the end of two months after the application of load increment? How many months will be required to reach a settlement of 7.5 cm? what is the settlement in 18 months? They had double drainage?

Degree of consolidation

$$U = \frac{e}{e_s} \times 100\%$$

$$t_1 =$$

$$t_2 = ?$$

Time factor's e
for $U < 60\%$



$$T_v = \frac{1}{4} \left(\frac{U}{100} \right)^2$$

For $U_1 = 20\%$ $(T_v)_1 = \frac{1}{4} (0.2)^2 = 0.0314$

For $U_2 = 50\%$ $(T_v)_2 = \frac{1}{4} (0.5)^2 = 0.1963$

$$\frac{t_2}{t_1} = \frac{(T_v)_2}{(T_v)_1} = \frac{0.1963}{0.0314}$$

$$t_2 = \frac{0.1963 \times 60}{0.0314} = 375 \text{ days} = 12.5 \text{ months}$$

$$\frac{T_v}{T_v} = \frac{t}{t_1}$$

$$T_v = \frac{t}{t_1} (T_v)_1 = \frac{18/2}{0.0314} \times 0.0314$$

$$T_v = 0.2826$$

$$U = 60\% \quad T_v = \frac{\pi}{4} (0.6)^2 = 0.2826$$

approximate expression is valid.

$$U = \frac{p}{p_f} \times 100$$

$$60 = \frac{p}{p_f} \times 100$$

$$p = 60\%$$



The time of reach drained laboura
Soil sample is 35

two way
bed clayey

Determine the time required for 60% consolidation of the same soil 10m thick on the top of a rocky surface subjected to the same loading conditions as the laboratory samples.

consolidation is less than 60% in both the cases.

$$T_1 = \frac{\pi}{4} \left(\frac{40}{100} \right)^2 = 0.785 \times 0.16 = 0.1256$$

$$T_2 = \frac{\pi}{4} \left(\frac{60}{100} \right)^2 = 0.785 \times 0.36 = 0.2826$$

For double drainage

$$T = C_v \frac{d^2}{E}$$

$$C_v = T_v \frac{d^2}{E}$$

$$T_v = \frac{C_v \cdot t}{d^2}$$

$$T_1 = \frac{C_v \cdot t}{d^2}$$

$$0.1256 = \frac{4 C_v \cdot t}{d^2}$$

$$0.1256 = \frac{4 C_v \cdot 35}{(100)^2} \rightarrow \textcircled{1}$$

For single drain $T_2 = \frac{C_v t}{d^2}$ $d = b$

$$0.28$$

$$\frac{\textcircled{2}}{\textcircled{1}} = \frac{0.2}{0.1256}$$

$$2.25$$



$$\frac{2.25}{0.1256} = \frac{C_v t \cdot 0.01}{100 \times 4 \times C_v \times 35}$$

$$t = \frac{0.2826 \times 4 \times 35 \times 100}{0.1256 \times (0.01)^2 \times 35}$$

$$t = \frac{0.2826 \times 4 \times 35 \times 100}{0.1256 \times (0.01)^2 \times 60 \times 60}$$

$$87500 \text{ hrs.}$$

$$= 3645.83 \text{ days.}$$

A clay layer of clay is located two layers of free draining sand. Also there is a thin drainage layer within the clay at a depth of 1.5m from its top surface. The average value of C_v is found as $4.92 \times 10^{-2} \text{ mm}^2/\text{s}$. If a structure is constructed above the clay layer. how many days would be required for it to obtain half the ultimate settlement. Assume that the expression $T = \pi/4 U^2$ is applicable for the entire range of consolidation.



$$T_{v1} = \pi/4 U_1^2 \quad \text{--- (1)}$$

$$T_{v2} = \pi/4 U_2^2 \quad \text{--- (2)}$$

$$T_{v1} = \frac{C_v}{d_1^2} t = \frac{4.92 \times 10^{-2}}{(150/2)^2} t$$

$$T_{v1} = 8.74 \times 10^{-4} \times 10^{-4} t = 8.74 \times 10^{-8} t$$

$$T_{v2} = \frac{C_v}{d_2^2} t = \frac{4.92 \times 10^{-2}}{(225)^2} t$$

$$T_{v2} = 1.71 \times 10^{-9} t$$

$$\frac{\pi}{4} U_1^2 = 8.747 \times 10^{-8} \text{ t}$$

$$\frac{\pi}{4} U_2^2 = 9.7185 \times 10^{-9} \text{ t}$$

$$\frac{U_1^2}{U_2^2} = \frac{8.747 \times 10^{-8} \text{ t}}{9.7185 \times 10^{-9} \text{ t}}$$

$$\frac{U_1^2}{U_2^2} = 9 \quad \therefore \frac{U_1}{U_2} = 3$$

$$\Delta H = U \cdot H$$

$$\Delta H_1 = U_1 \cdot H_1$$

$$\Delta H_1 = U_1 \times 1.5$$

$$\Delta H_2 = U_2 \times 4.5 = 4.5 U_2$$

$$\Delta H = \Delta H_1 + \Delta H_2$$

$$1.5 (3U_2) + 4.5 U_2$$

$$1.5 \times (3 \times U_2)$$

$$U_2^2 =$$



$$\frac{\pi}{4} U_2^2 = 9.7186 \times 10^{-9} \text{ t}$$

$$\frac{\pi}{4} \left(\frac{1}{3}\right)^2 = 9.7185 \times 10^{-9} \text{ t}$$

$$\frac{\pi}{4} \times \frac{1}{9} = 9.7185 \times 10^{-9} \text{ t}$$

$$t = \frac{0.08722 \times 10^{-3}}{10^{-9}} = 0.08722 \times 10^6$$

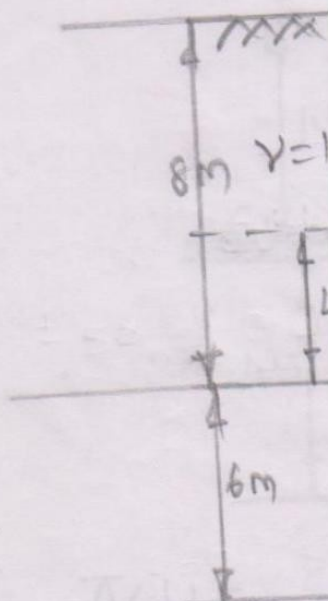
$$= \frac{8.97 \times 10^{-3}}{10^{-9}}$$

$$= 8.97 \times 10^6 \text{ sec}$$

$$t = 103.93 \text{ days}$$

A stratum of clay with an average liquid limit of 45% is 6m thick. The surface is located at a depth of 8m below the ground surface. The nature of clay is 40% & the S.G is 2.7. Between ground surface & clay with the subsoil consists of fine sand.

The water is located at a depth of 4m below the ground surface. The average submerged unit wt of sand is 10.5 kN/m^3 & unit wt of sand above the H_2O table is 17 kN/m^3 . The wt of building that will be constructed on the sand above clay increases the overburden pressure on the clay by 40 kN/m^2 . Estimate the settlement of building.



clay
 $w_L = 45\%$
 $w = 40\%$
 $G = 2.7$

Pressure on the top of Clay due to overburden = σ

$$\gamma \times h + \gamma' \times h = 4 \times 17 + 4 \times 10.5$$

$$\sigma' = 110 \text{ kN/m}^2$$

Increase in pressure due to consolidation of building
 $= 40 \text{ kN/m}^2 = \Delta \sigma$

$$\text{Settlement} = \frac{C_c H}{1 + e} \log_{10} \frac{\sigma' + \Delta \sigma}{\sigma'}$$

$$\Rightarrow 0.315 \times$$

$$= \frac{C_c \times 6}{1 + e_0} \log_{10} \frac{\sigma'_1 + \Delta \sigma}{\sigma'_1}$$

$$S_r e = w G$$

$$e_0 = w_{sat} G$$

$$C_c = 0.009 (w_L - 10)$$

$$C_c = 0.009 (45 - 10) = 0.315$$

$$\text{Settlement } S = \frac{C_c H}{1 + e_0} \log_{10} \frac{\sigma'_1 + \Delta \sigma}{\sigma'_1}$$

$$= \frac{0.315 \times 6}{1 + 1.08} \log_{10} \left(\frac{110 + 40}{110} \right)$$

$$S = 0.1224 \text{ m}$$

A saturated soil stratum lies above an impervious stratum. It has a coefficient of permeability of $3.5 \times 10^{-4} \text{ cm/sec}$ at a stress of 210 kN/m^2 . Determine the change in void ratio.



$$\Delta e = C_c \cdot w \log_{10} \frac{\sigma'_2}{\sigma'_1}$$

$$= 0.28 \times \log_{10} \frac{210}{750} = 0.0409 \text{ (decrease)}$$

(i) Settlement of soil stratum

$$\frac{\Delta H}{H_0} = \frac{\Delta e}{1 + e_0}$$

$$\Delta H = \frac{\Delta e \times H_0}{1 + e_0} = \frac{6 \times 0.0409}{1 + 1.95} = 0.0832 \text{ m}$$

(ii) Time required for 50% consolidation

$$t_{50} = \frac{T_v \cdot d^2}{c_v} \quad d = 6 \text{ m for single drainage}$$

$$C_v = \frac{k}{m_v \gamma_w}$$

$$m_v = -\frac{\Delta e}{1+e_0} \cdot \frac{1}{\Delta \sigma'}$$

$$= -\frac{\Delta e}{\Delta \sigma'} \cdot \frac{1}{1+e_0}$$

$$m_v = a_v \cdot \frac{1}{1+e_0}$$

$$C_v = \frac{(1+e_0)}{a_v \cdot \gamma_w}$$

$$C_v \Delta \sigma' = \frac{k (1+e_0) \Delta \sigma'}{a_v \cdot \gamma_w}$$

$$m_v = 3$$

$$C_v =$$

$$\pm 50 = 0.2$$

$$= 4.663 \times 10^{-3}$$

$$\pm 50 = 4663 \text{ sec.}$$



A layer of clay 2m thick is subjected to a loading of 0.5 kg/cm². One year after loading, the average consolidation is 50%. The layer has double drainage, (i) what is the coefficient of consolidation? (ii) if the coefficient of permeability is 3 mm/year, what is the settlement after one year & how much time will the layer take to reach 90% consolidation?

$$H = 2\text{m} \quad d = 2/2 = 1\text{m}$$

$$U = 50\% \quad t = 1\text{year} \quad \Delta \sigma = 0.5\text{kg/cm}^2 = 0.5 \text{ m.w}$$

(a) Determination of C_v : $U < 60\%$

$$T_{v1} = \pi/4 \times U_1^2 = \pi/4 (0.5)^2 = 0.1963$$

$$C_v = T_{v1} \cdot d^2/6 = 0.1963 \times 1^2/1 = 0.1963 \text{ m}^2/\text{year}$$

(b) Determination of Settlement after 1 years.

$$m_v = \frac{k_v}{c_v \cdot r_w} = \frac{3 \times 10^3 \text{ m/yr}}{0.1963 \text{ m}^2/\text{year} \times 1000 \text{ kg/m}^3}$$

$$P_s = m_v \cdot \Delta \sigma$$

$$10^4$$

$$= 0$$

Settlement after

$$= 0.0$$



$$0.1963 \text{ m}^2/\text{kg} \times 0.5 \times$$

$$= 0.1528 \times 0.5$$

(c) Time taken for 90% Settlement

$$U_2 > 90\% > 60\%$$

$$T_{v2} = -0.9332 \log_{10} (1 - 0.9) = 0.851$$

Determination of coefficient of consolidation (C_v)

The recorded U vs t changes during one of the load stages in an oedometer test are used to evaluate the coefficient of consolidation (C_v). The process involves plotting the changes against time (O time or log time) & then fitting to this to the

theoretical T_v curve.

In this way

U are located

Two methods

① Square root

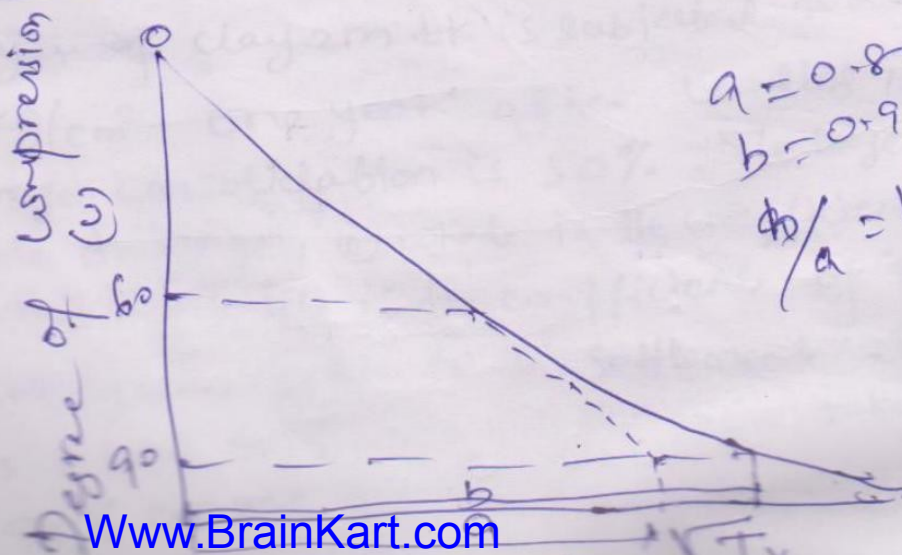
② Logarithm of

Square root of time fitting method



of T_v ,
may be located

method
method



$$a = 0.80$$

$$b = 0.92$$

$$\frac{b}{a} = 1.15$$

Characteristic Curve b/w $\sqrt{T_v}$ & U . The curve is straight upto $U=60\%$ &

when $U=90\%$ then the distance to be equal to 1.15 times of $U=60\%$.

Taylor suggested that the characteristic of the theoretic curve to determine the 90% U point on a labour curve.

coefficient of consolidation

$$C_v = \frac{T_{90} H^2}{H^2}$$

Primary con

The log time



many compression
tal Compression.

An A

time fitting method.

this is widely useful when there is significant

Secondary compression. Casagrande & RE

Fadum (1939) devised this method

Consolidation curve is plotted on a semi logarithmic scale with time factor on logarithmic scale & degree of consolidation on arithmetic scale. Tangent of the tangent & asymptote is at the ordinate of 100%.

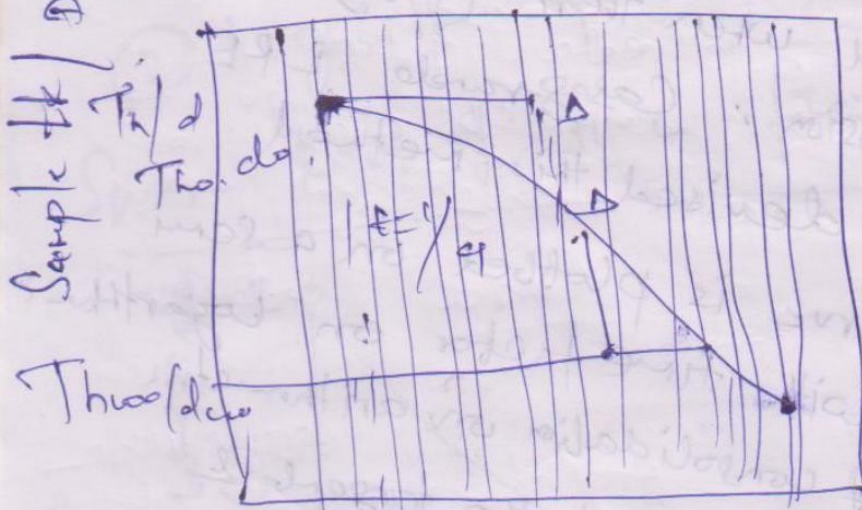
$$C_v = \frac{T_{50} H^2}{t_{50}}$$



Theoretical
Degree

Labouratory

Dial gauge
readings



Sample thickness / Dial gauge readings
Time to number (log scale) . log of scale
Labouratory curve

SRI VIDYA COLLEGE OF ENGINEERING & TECHNOLOGY

VIRUDHUNAGAR

DEPARTMENT OF CIVIL ENGINEERING

CE6405-MECHANICS OF SOILS

UNIT-IV

SHEAR STRENGTH



BY

Mr.R.PANDIARAJAN/AP/CIVIL

Shear Strength

Shear Strength of cohesive & cohesionless soil
 - Mohr - Coulomb failure theory Saturated
 Soil - Strength parameters - Measurement of Shear
 Strength - direct shear, Triaxial compression, UCC -
 & Vane shear tests - Types of ^{shear} tests based on
 drainage & their applicability - Drained &
 undrained behaviour of clay & sand -
 Stress path for conventional triaxial tests.

* Shear Strength of soil

Shear strength is the property of soil that describes its ability to resist shear stress.



in soil mechanics shear stress is the stress that tends to cause one part of the soil to move relative to another part. It is represented by the symbol τ . Shear stress is a type of stress that acts parallel to the surface of the soil. It is caused by forces that are applied to the soil in a direction that is parallel to the surface. Shear stress is a measure of the soil's resistance to deformation. It is a key parameter in soil mechanics and is used to determine the stability of soil structures.

* Measurement of

The measurement of shear strength of soil involves certain tests observations at failure with the help of which the failure envelope.

Shear resistance of soil can be determined in the laboratory by the following four methods

- ① Direct shear test
- ② Triaxial shear test
- ③ Unconfined compression test
- ④ Vane shear test

Depending upon the Drainage condition three types of tests are developed.

(a) Undrained test (i) quick test

Drainage is not permitted at any stage of the test that is either before the test during the application of the normal stress. Usually this test takes only 5 to 10 minutes performed only for soils of low permeability.

(b) Consolidated undrained test : (i) consolidated (ii) undrained (iii) quick test

Drainage is permitted fully in this type of test during the application of normal stress & no drain during the test. It is permitted after stress. 5 to 10 minutes.



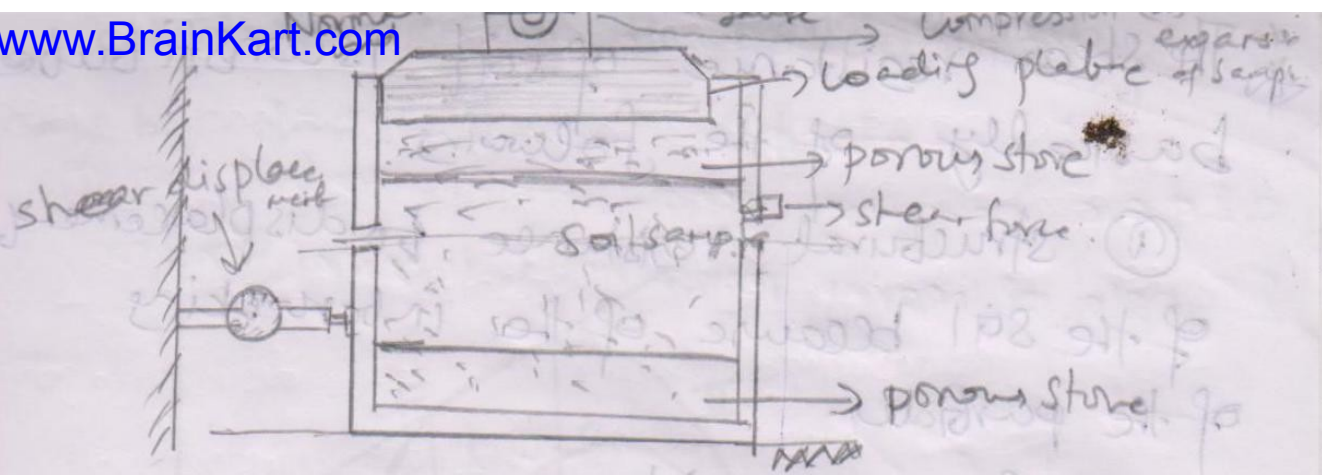
(c) Drained test

Drainage is permitted before & during the test during the application of both normal & shear stresses. No excess pore pressure develops at any stage. It may take 4 to 6 weeks to complete. Cohesive soils.

Laboratory:

(1) Direct shear test

Shear box apparatus essentially consists of brass box, split horizontally at middle of the soil specimen.



The soil is gripped in perforated metal grilles, behind which porous discs can be placed if required to allow the specimen to drain. Normal load is applied on the specimen from a loading gauge. Usually the plan size of the

specimen is 60 mm square. Testing large gravel, $h_b = 2$ upper half



large size employed for serial such as is applied zero initially

σ is increased under

Two types of application of shear are possible. one \rightarrow shear stress controlled, shear strain is controlled. shear stress controlled type load is applied at constant rate or more commonly in equal increments at means of calibrated w_b . Shear displacement is measured with the aid of a dial gauge attached to the side box.

In strain controlled type shear displacement is applied at a constant rate by means of a screw operated manually or by motor of the universal rig.

→ The shear resistance of soil is constituted basically of the following

① structural resistance to displacement of the soil because of the interlocking of the particles.

② The frictional resistance.

③ Cohesion or adhesion b/w the surface of the soil particles.

Coulomb's law:

$$S = C + \sigma \tan \phi$$

where c - cohesion

S - shear stress



slope of the right line of above equation.
angle of internal friction @ angle of shearing resistance conditions

c & ϕ are not

Soil. They depend on the

Strength theories for soils: No of theories have been propounded for explaining the shear strength of soil.

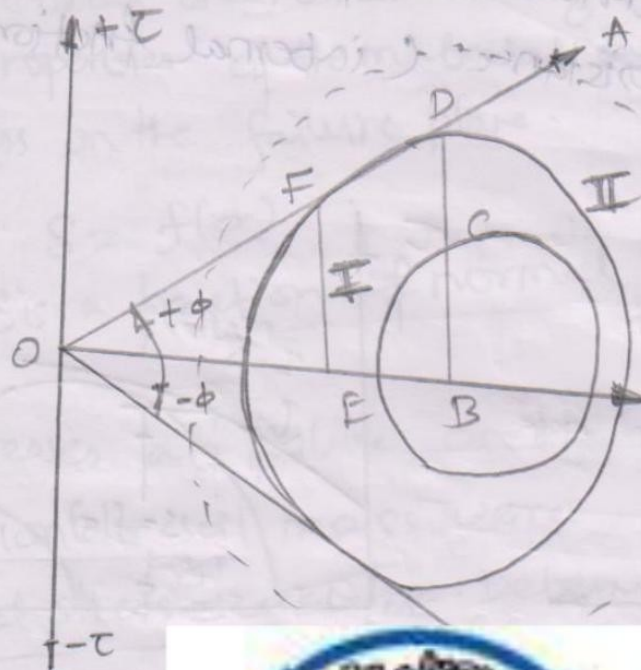
Mohr - Coulomb failure theory: 2 Mohr

Strength theory are a generalisation & modification of the Coulomb's equation.

Mohr: Strength theory:

shearing stress may be expressed as $\tau = \sigma \tan \phi$ on any plane. where ϕ is the angle of obliquity. angle \uparrow @ has limiting value.

Soil of the shearing strength may be $S = c + \sigma \tan \phi$.



Mohr's strength

stress condition

Mohr's circle

BC \rightarrow shear

OD \Rightarrow Normal

soil

presented by
D = shearing strength
or this normal stress
is $>$ than BC

stress condition as represented by the Mohr's circle II which tangential to the Mohr's envelope as if stress condition is impossible to apply to the Mohr's circle III dashed to this soil sample.

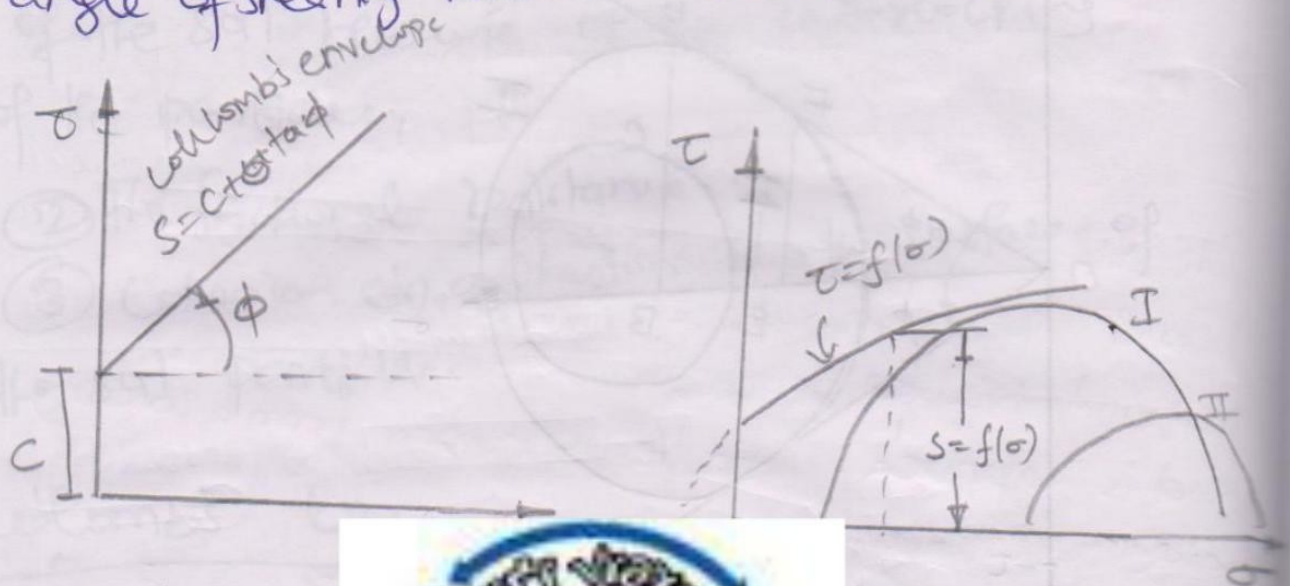
Mohr's Coulomb theory:

First propounded by Coulomb (1776) & later generalised by Mohr it is most commonly used concept. The functional relationship between the normal stress on any plane & the shearing strength was assumed to be linear

Usually known as Coulomb's law: $\tau = c + \sigma \tan \phi$

$$\tau = c + \sigma \tan \phi$$

c & ϕ are parameter known as apparent cohesion & angle of shearing resistance (internal friction)



(a) Coulomb's envelope

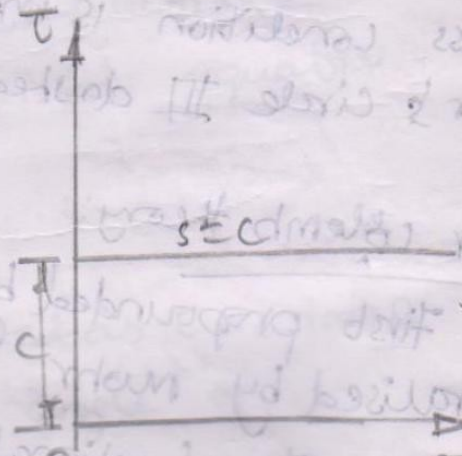
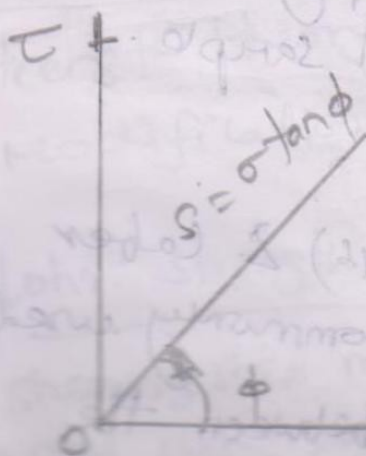
Mohr
Mohr generalised
as a curve which
failure envelope



is generalised
failure envelope
envelope (or)
increase in normal
stress.

The Coulomb envelope

the shapes for purely cohesionless (or) granular
soil (or) pure sand.



Mohr's theory Assumptions:

- * Material fails essentially by shear.
- * critical shear stress causing failure depends upon the properties of the material as well as normal stress on the failure plane.

$$S = f(\sigma)$$

S is a function of normal stress.

The stresses at failure on the failure plane in a cohesionless soil mass were shear stress = 4 kN/m^2 .
Normal stress $\sigma = 10 \text{ kN/m}^2$. Determine the resultant angle of internal friction of the failure plane to the horizontal.



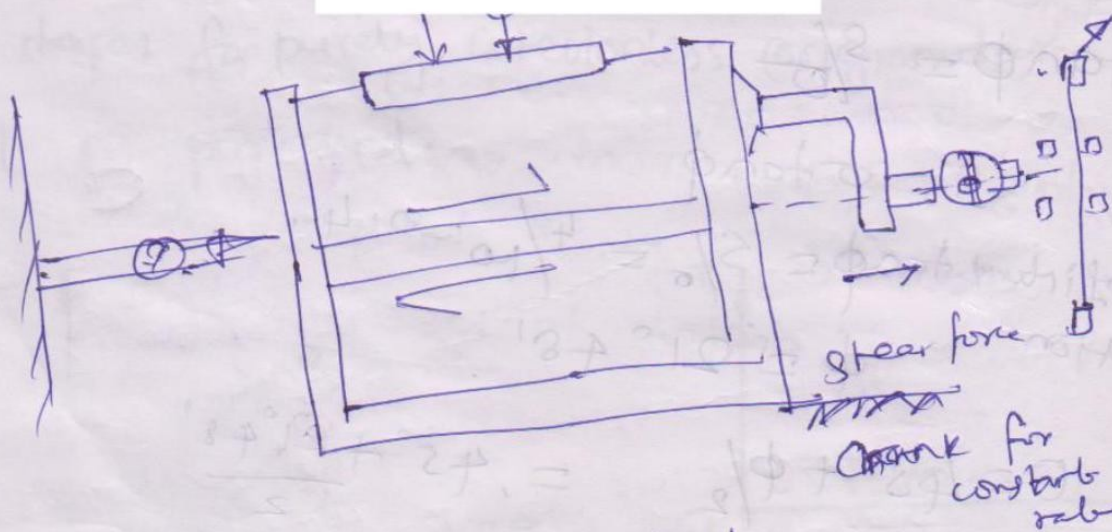
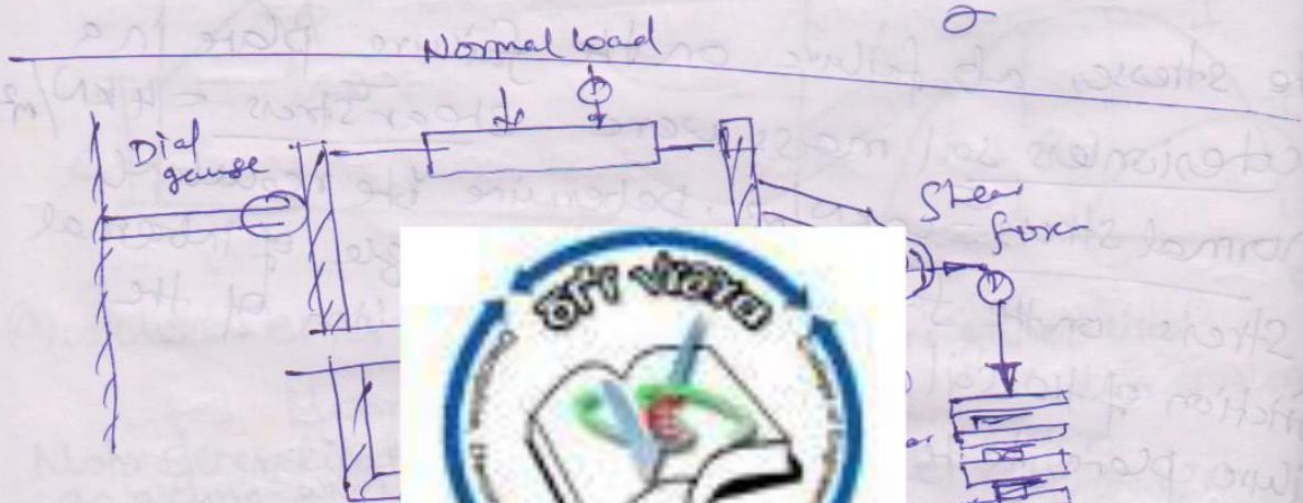
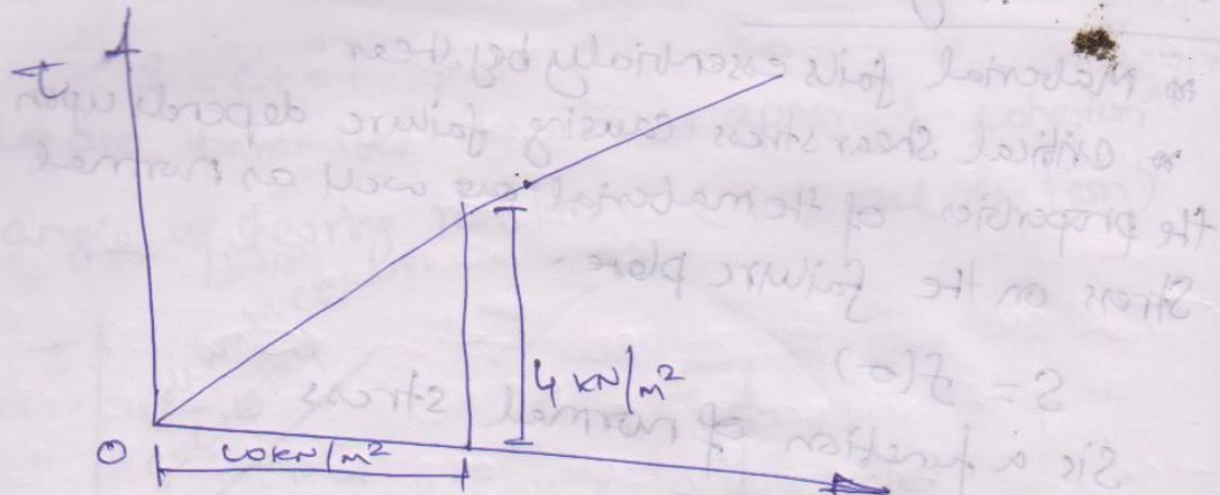
$$\tan \phi = S/\sigma$$

$$S = \sigma \tan \phi$$

angle of internal friction $\tan \phi = S/\sigma = 4/10 = 0.4$
 $\phi = 21^\circ 48'$

angle of inclination $\theta = 45^\circ + \phi/2 = 45^\circ + \frac{21^\circ 48'}{2}$
 $= 55^\circ 54'$

Graphical Solution



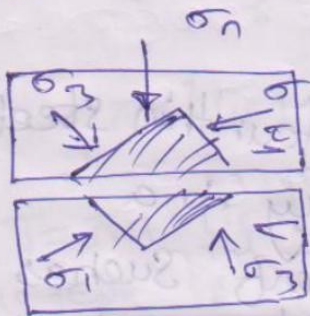
widely used.

Strain Control

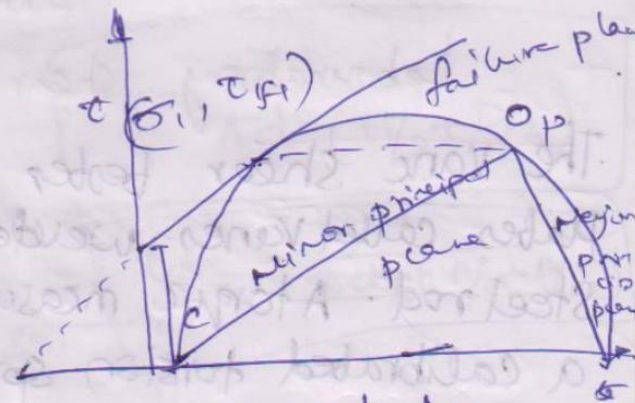
Direct shear device.

Both cases normal load is applied to which the normal load

ratio of the shear displacement to the thickness of the sample.
The Mohr circle representation of stress conditions in direct shear test.



Conditions of stress in the shear box



Mohr's circle for d. s. t.

Simple test.

Disadvantages

As Triaxial compression

- The distribution is not uniform
- not used
- There is side walls



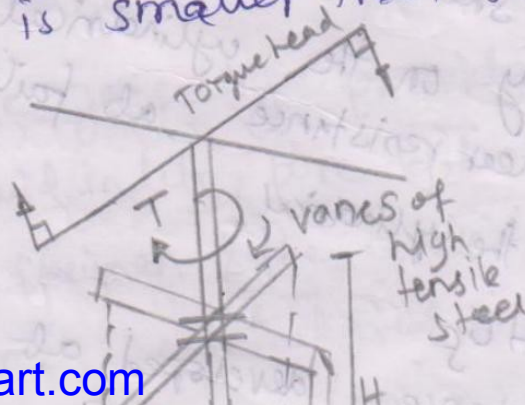
control on the drainage of soil.

zero stress is

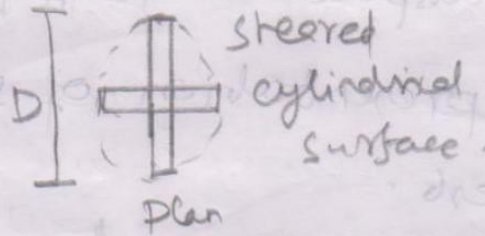
maintained by the

VANE SHEAR TEST

It is a quick test used either in laboratory or in the field. to find out the undrained shear strength of cohesive soil. Laboratory Vane Shear Apparatus is smaller than field vane shear apparatus.



Lab	
H	= 20 mm
D	= 12 mm
L	= 0.5 to 1 mm
Field	
H	= 10 to 20 cm
D	= 5 to 10 cm



Labouratory Shear vane

The vane shear tester consists of four thin steel plates called vanes welded orthogonally to a steel rod. A torque measuring arrangement such as a calibrated torsion spring is attached to rod which is rotated by a worm gear & worm wheel arrangement.

After pushing the vanes gently into the soil, the torque rod is rotated at a speed (usually about 1° per min). The deflection is indicated by a dial attached to the rod. The torque T is then read from the dial reading.



C_u = unit strength of soil

H = height of the vane.

D = diameter of the vane.

When the top end of the vane is embedded in the soil, so that the both top & bottom ends partake in the shearing of the soil.

Assume that the shear resistance of the soil is uniformly on the cylindrical surface at failure.

where $2\pi r dr \tau_f$.

Total resistance of both top & bottom faces will be

$$= 2 \int_0^{\pi} (2\pi r dr) \tau_f$$

$$T = \pi d^2 \tau_f \left(\frac{H}{2} + \frac{D}{6} \right)$$

$$T = \pi d^2 \tau_f \left[\frac{H}{2} + \frac{D}{12} \right]$$

This test is suited for clayey soil

$$\tau_f = \frac{T}{\pi d^2 \left(\frac{H}{2} + \frac{D}{6} \right)}$$

$$\tau_f = \frac{T}{\pi d^2 \left(\frac{H}{2} + \frac{D}{12} \right)}$$

Triaxial Compression Test:

This test was introduced by Casagrande & Terzaghi in 1936, most popular test used in field & for the purpose of specimen is subjected to mutually + d increases

a cylindrical specimen diameter is the major principal direction & c



Suggests the soil compressive stresses in the specimen shear. Usually equal to twice its axial cylindrical test applied in the vertical axis σ_2 & σ_3 are by the fluid

applied in the horizontal direction, pressure round the specimen.

The test equipment specimen consists of a high pressure cylindrical cell made of Perspex or other transparent material fixed b/w the base & the top cap. Three outlets connections are generally provided through the base : cell fluid inlet, pore H₂O outlet from the bottom of the specimen & the drainage outlet from the cylindrical specimen is dependent upon

drainage conditions of the test, solid nonporous disc or end caps or porous disc are placed on the top & bottom of the specimen.

Test procedure:

- ① A saturated porous stone is placed on the pedestal & the cylindrical soil specimen is placed on it.
- ② The specimen is enveloped by a rubber membrane to isolate it from water with which the cell is filled. The cell is sealed with the pedestal with rubber "O" rings.
- ③ Cell is filled with H_2O . The soil specimen is subjected to the cell pressure is called confining pressure.
- ④ Axial stress is continuously applied until failure of the specimen occurs.



Number of observation:

- ① As the cell pressure is applied pore H_2O pressure develops in the specimen, which can be measured with the help of a pore pressure measurement device or Bishop's pore

(ii) Pore pressure is to be dissipated, pore water line is closed, the drainage line opened & connected to the bubbler.

(iii) The axial strain associated with the app of downward axial stress can be measured by means of a dial gauge, set to record the downward movement of a loading piston.

Area Correction for the Determination of Additional Axial Stress (a) Deviatoric Stress.

* The addition of any stress at any stage of the test is determined by the difference between the applied axial compressive stress and the initial axial stress.



applied at any stage of the test is determined by the difference between the applied axial compressive stress and the initial axial stress.

A_0, h_0 & V_0 are the initial area of cross section, height & volume of the soil specimen respectively. If A, h & V are the corresponding values at any stage of the test, the corresponding changes in the values being $\Delta A, \Delta h$ & ΔV then

$$A(h_0 + \Delta h) = V = V_0 + \Delta V$$

$$A = \frac{V_0 + \Delta V}{h_0 + \Delta h}$$

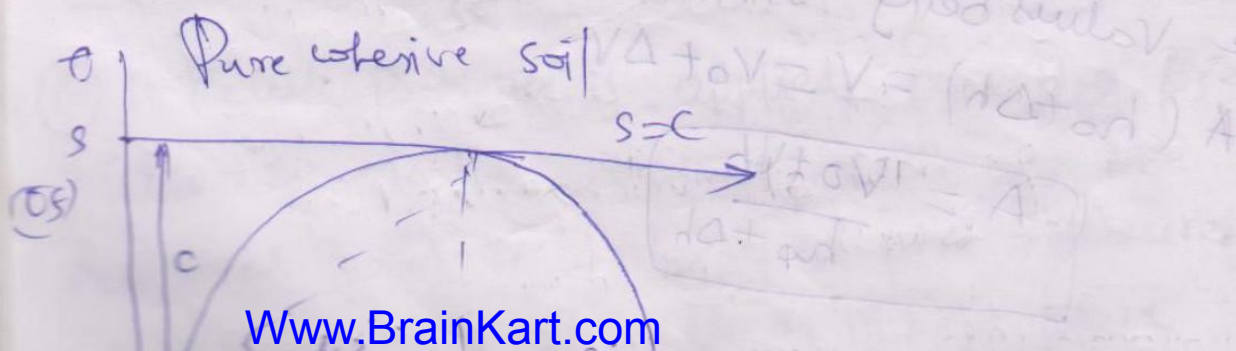
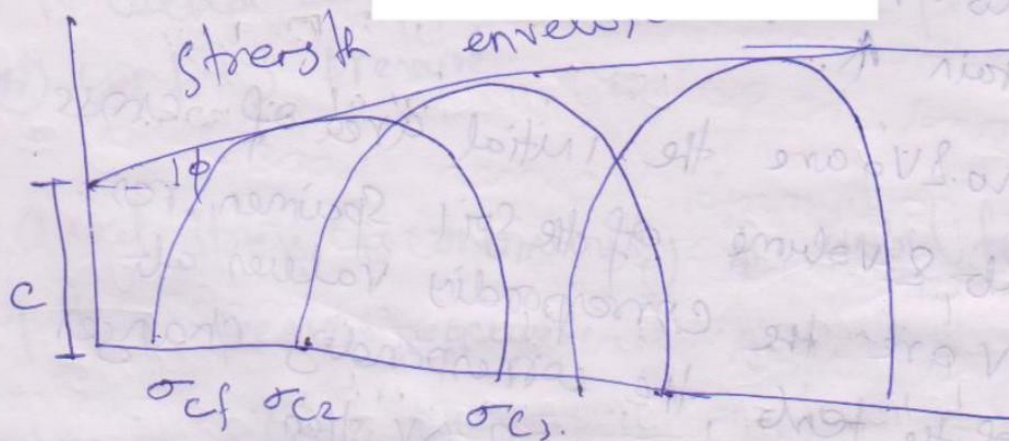
$$A = \frac{V_0 + \Delta V}{h_0 - \Delta h} = \frac{V_0 \left(1 + \frac{\Delta V}{V_0}\right)}{h_0 \left(1 - \frac{\Delta h}{h_0}\right)}$$

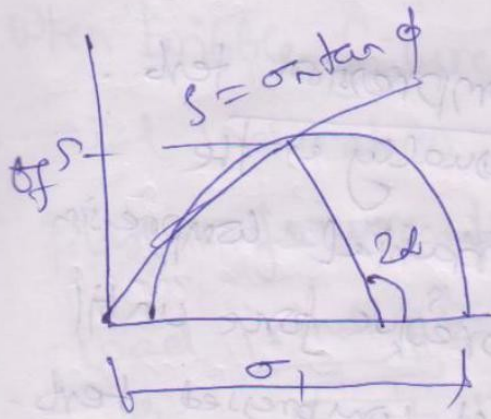
$$= A_0 \left(1 + \frac{\Delta V}{V_0}\right) \frac{\left(\frac{\Delta h}{h_0}\right)}{(1 - \epsilon_a)}$$

Mohr circle for Triaxial test.



Different cases





$$N\phi = \tan^2 \alpha = \tan^2 (45^\circ + \phi/2)$$

Merits:

* Shear tests under all the three drainage conditions are applied

* Accurate volume change

* Stress distribution

* Extension

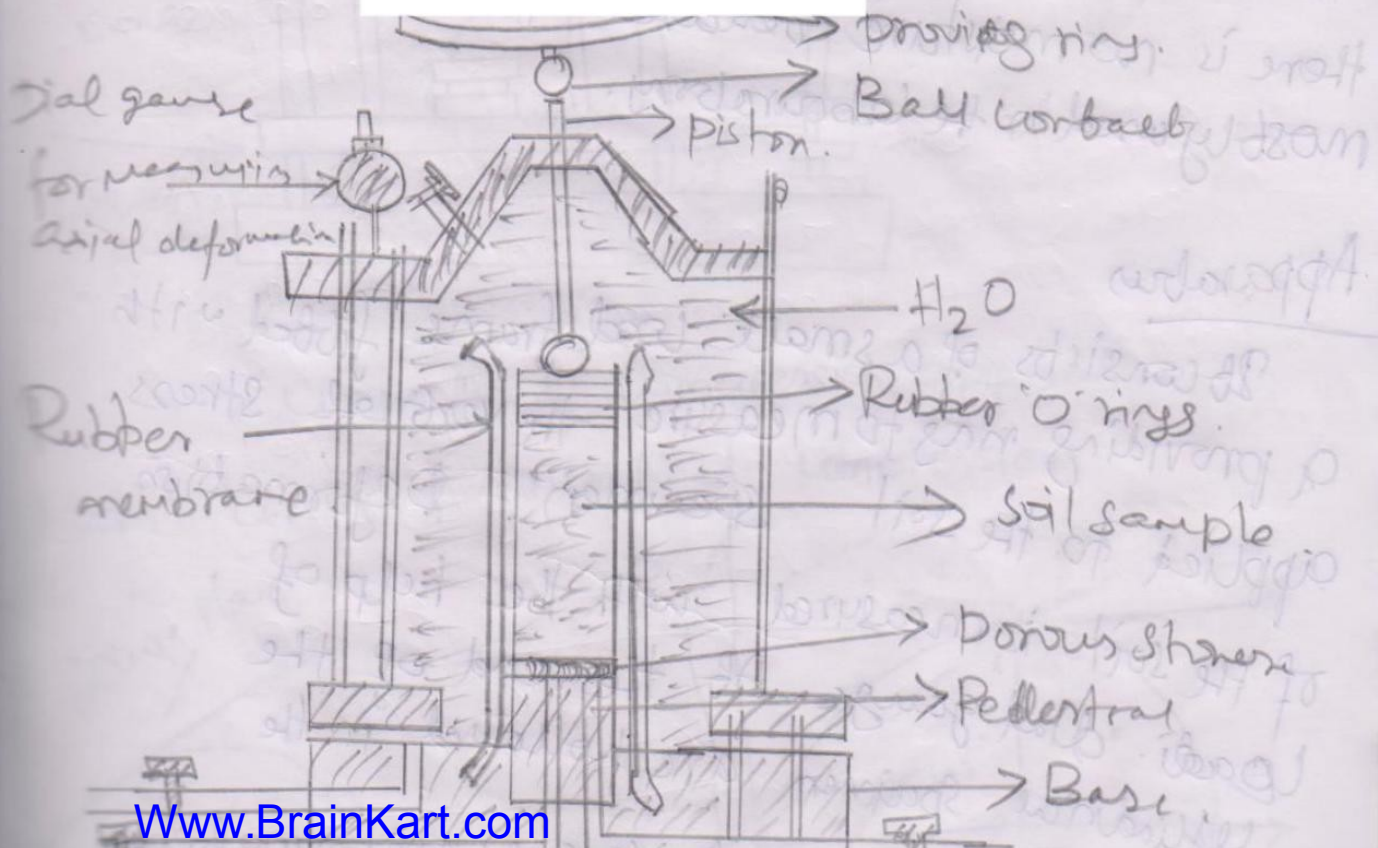
with the



ore pressure & Mr are possible.

to be concluded

exclus.



Unconfined Compression test:

Special case of triaxial compression test.
A cylindrical soil specimen usually of the same size as that for the triaxial compression is loaded axially by compressive force until failure takes place. Unconfined compression test. No need of the rubber membrane. axial stress is the major principal stress and other two principal stresses are zero.

This test is
(or) remoulded
for coarse grained

This test is known
there is no moisture
mostly used in the laboratory.



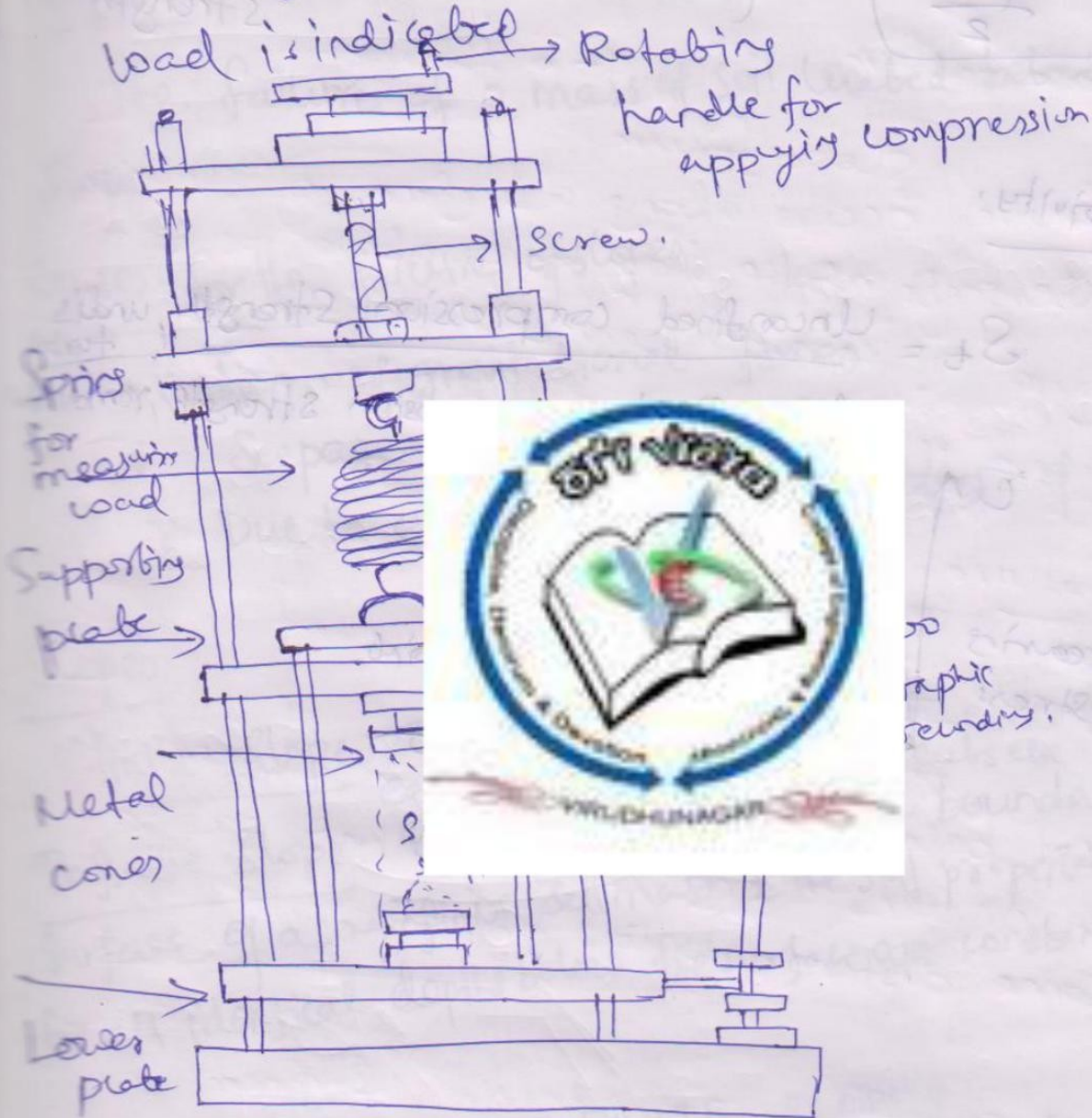
on undisturbed
ob be conducted
red & gravels.
because
his test.

Apparatus

It consists of a small load frame fitted with a providing ring to measure the vertical stress applied to the soil specimen. Deformation of the soil is measured with the help of load cell dial gauge. At the end of the test, the specimen are hollowed in the

Readings are taken & graph is plotted.

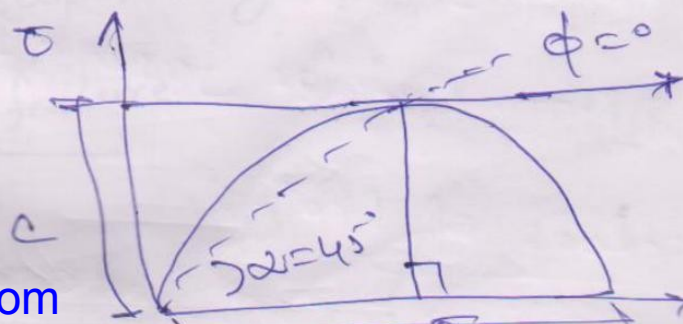
When brittle failure occurs the providing ring dial indicates a definite maximum load. Plastic failure there is no definite maximum load is indicated.



Unconfined compression apparatus.

Mohr Circle for unconfined compression

Failure plane.



$$\sigma_1 = 2c \tan(45^\circ + \phi/2)$$

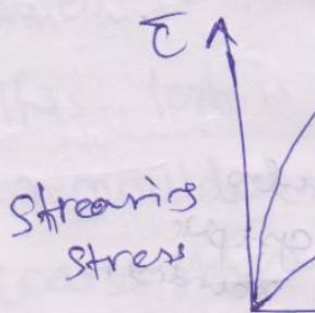
$$\sigma_1 = \phi_u = 2cu$$

$$cu = \frac{\phi_u}{2}$$

ϕ_u = unconfined compression strength

Sensitivity.

$S_t = \frac{\text{Unconfined compression strength, undisturbed}}{\text{Unconfined compression strength, remoulded}}$



SRI VIDYA COLLEGE OF ENGINEERING & TECHNOLOGY

VIRUDHUNAGAR

DEPARTMENT OF CIVIL ENGINEERING

CE6405-MECHANICS OF SOILS

UNIT-V

FLEXIBILITY OF SLOPES



BY

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UNIT: 5. STABILITY OF SLOPES

Earth embankments are commonly required for railways, roadways, earth dams, levees & river training works.



Slide:

The failure of a mass of soil located below a slope.

Causes for the failure of slopes:

- * action of gravitational forces.
- * Seepage
- * Due to



Slopes:

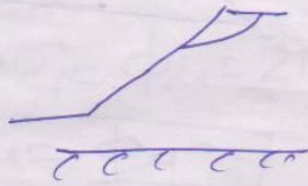
Finite Slope

Infinite Slope

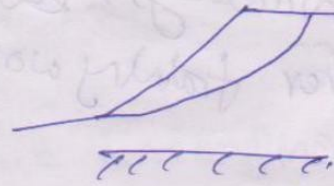
Surface of a semi-infinite soil mass & the soil properties for all identical depths below the surface are constant.

STABILITY ANALYSIS OF FINITE SLOPES

Failure of finite slopes occurs along a surface which is a curve. Two basic types of failure of a finite slope may occur: (i) Slope failure (ii) base failure



a) face failure



toe failure.

Slope failure.

If the failure occurs along a surface of sliding that intersects at the slope at or above its toe face failure. If the arc passes above the toe or toe failure if the arc passes through the toe.

Base failure



If the soil below the failure occurs

at some distance below the toe of the slope.

Types of Slip Surfaces (or) failure Surfaces:

- a finite slope may take place along one of the following the rupture of failure surface
- 1) Planar failure surface
 - 2) circular failure surface
 - 3) Non circular failure surface.

Methods of Analysis:

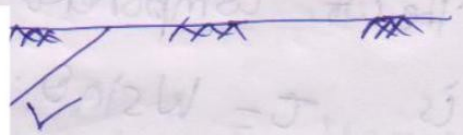
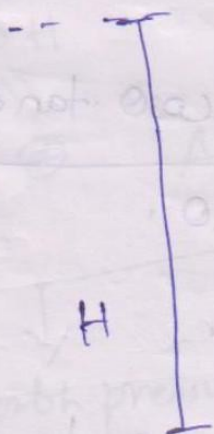
The stability of finite slope can be investigated

- 1) Culmann's method of plane failure surface
- 2) Swedish circle method (slip circle method)
- 3) friction circle method
- 4) Bishop's method.

Planar failure Surface: Culmann's method

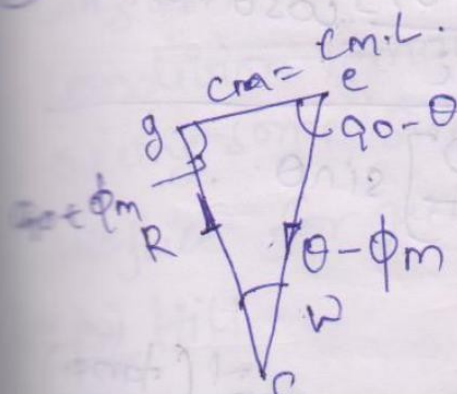
Culmann (1866) considered a simple failure mechanism of a slope of homogeneous soil with plane failure surface passing through the toe of the slope.

AB → probable slip surface.
 ABB wedge is in equilibrium under the action of three forces.



Culmann's slip plane

- ① wt of the wedge, $W = \frac{1}{2} AB \cdot h \cdot \gamma = \frac{1}{2} L \cdot h \cdot \gamma \rightarrow \text{①}$
- ② The cohesive force c along the surface AB, resisting motion $= c \cdot m \cdot L$.
- ③ The reaction R inclined at an angle ϕ_m to the normal



$$h = \frac{H \sin(i-\theta)}{\sin i}$$

Sub in (1) we get $W = \frac{1}{2} \cdot L \cdot \gamma \frac{H \sin(i-\theta)}{\sin i}$

If c & ϕ are the app shear strength parameters

$$\tau_f = c + W \cos \theta \cdot \tan \phi$$

The wt component $W \sin \theta$ to the plane AC causing sliding

is $\tau = W \sin \theta$

Factor of safety



$$F = \frac{c + W \cos \theta \cdot \tan \phi}{W \sin \theta}$$

Sub the value of

$$F = \frac{c + \frac{1}{2} \gamma H \left[\frac{\sin(i-\theta)}{\sin i} \right] \cos \theta \tan \phi}{\frac{1}{2} \gamma H \left[\frac{\sin(i-\theta)}{\sin i} \right] \sin \theta}$$

$$F = \frac{c + \frac{1}{2} \gamma H \left[\frac{\sin(i-\theta)}{\sin i} \right] \cos \theta \tan \phi}{\frac{1}{2} \gamma H \left[\frac{\sin(i-\theta)}{\sin i} \right] \sin \theta}$$

$$F = \frac{c + \frac{1}{2} \gamma H \left[\frac{\sin(i-\theta)}{\sin i} \right] \cos \theta \tan \phi}{\frac{1}{2} \gamma H \left[\frac{\sin(i-\theta)}{\sin i} \right] \sin \theta}$$

$c_m \Rightarrow$ mobilised cohesion $c_m = \frac{c}{F}$

$\phi_m \Rightarrow$ mobilised cohesion friction $= \tan^{-1} \left(\frac{\tan \phi}{F} \right)$

$$\frac{C_m L}{\sin(\theta - \phi_m)} = \frac{W}{\sin(90 + \phi_m)} = \frac{R}{\sin(90 - \theta)}$$

$$\frac{C_m L}{\sin(\theta - \phi_m)} = \frac{W}{\cos \phi_m} = \frac{W}{\sin(90 + \phi_m)} \quad [\because \sin(90 + \theta) = \cos \theta]$$

$$\frac{C_m L}{\sin(\theta - \phi_m)} = \frac{\frac{1}{2} L V H \sin(i - \theta)}{\sin i \cos \phi_m}$$

$$\frac{C_m}{V H} = \frac{1}{2} \left[\frac{\sin(\theta - \phi_m) \sin(i - \theta)}{\sin i \cos \phi_m} \right]$$



Earth pressure

Strength theory

Stability analysis of slope

Infinite slopes

Slope that extend over a long distance & the condition remain identical along some surface or surface for quite some distances
(ex) Hill

Finite slopes
Slope that has a limited extent

(ex) embankment, highways, railways embankment

It can be natural or manmade

Types of Slopes:

Natural

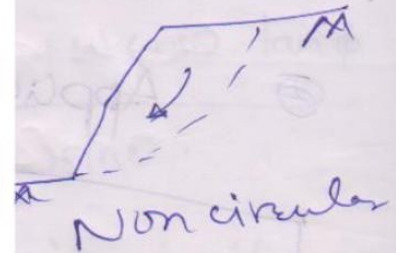
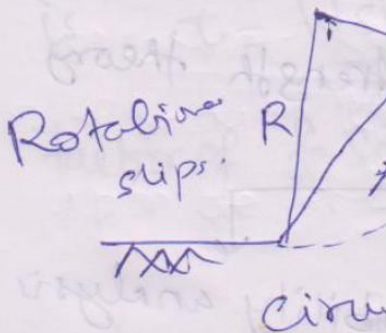
- Hill side & valleys
- * coastal & river cliffs

Manmade

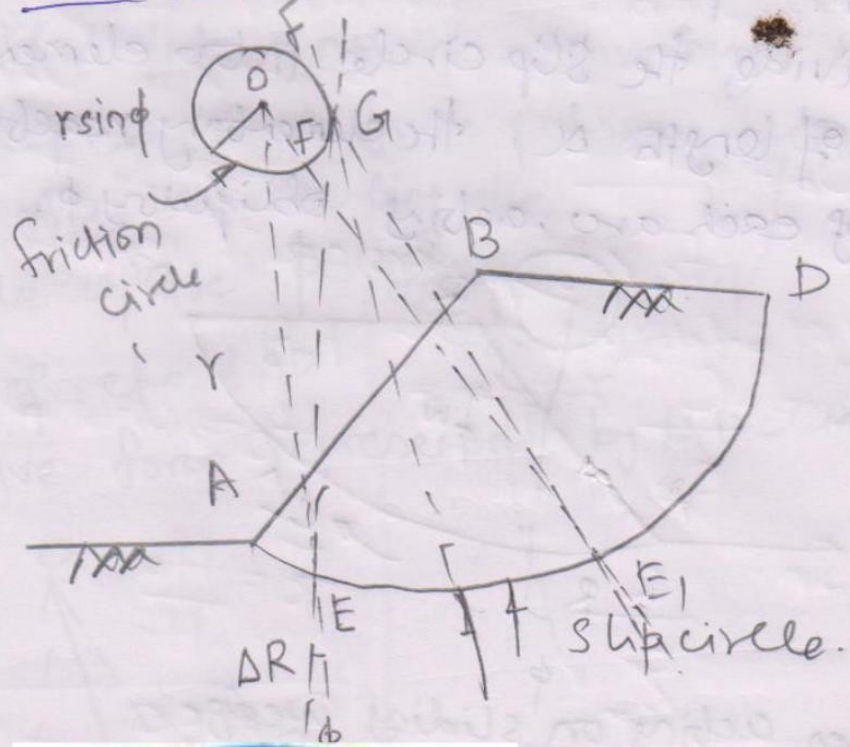
- * cutting & embankments for highways & rail road
- * Earth & ash pond dams
- * Temporary excavation

Types of slope failure:

Rotational



Friction Circle Method:



- * Friction circle is the arc of failure surface of rotation of a small circle with O as the center.



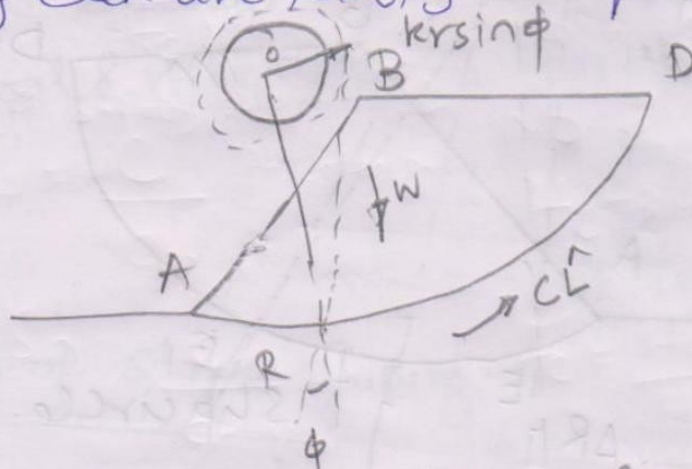
- * A line EF tangential to this smaller circle will cut the failure arc AD at an obliquity ϕ .

- * Conversely any vector representing reaction DR at an obliquity ϕ to an element of the failure arc AD will be tangential to the small circle. This small circle of radius $r \sin \phi \Rightarrow$ friction circle $\odot \phi$ circle.

Forces acting on the sliding wedge ABDA

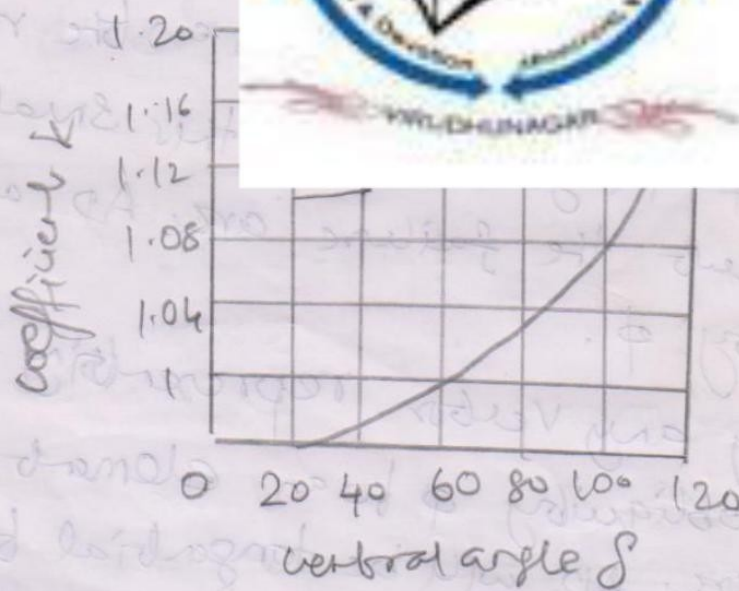
- * Total weight W to the wedge.

* Divide the slip circle into elementary arc of length ΔL , the elementary reaction ΔR of each arc, acting obliquely by ϕ .



Forces acting on sliding wedge.

concentric circle
of radius $kR \sin \phi$
upon the central



Value of coefficient k.

Let C_m = mobilised unit cohesion
assumed constant along the arc.
Mobilised cohesion on elementary arc
of length ΔL

Total cohesive resistance $c_m L = c_m \Delta L$
 the arc AD, divided into a no of elementary arc
 of length ΔL , represents a force polygon wedge
 & the chord AD, representing the
 closing side of the polygon,



\bar{L} = length of chord AD
 Total cohesive force represented by AD = $c_m \bar{L}$.



$$c_m L \cdot a = c_m \cdot r$$

$$L \cdot a = r$$

$$a = r$$

$$c_m L \cdot a = c_m \cdot r$$

$$a = \frac{r}{L}$$

$$\text{Mobilised cohesion} \times a (\text{radius}) = c_m \cdot L \cdot a = c_m \cdot r$$

$$c_m L a = c_m r$$

$$a = \frac{r}{L}$$

Factor of safety F_c with respect to cohesive strength

$$F_c = \frac{c}{c_m}$$

A no of slip circle are taken & factor of the circles given.

Taylor's stability Number &

7

stability curves

$$S_n = \frac{c}{F_c \gamma H}$$

Factor of safety with respect to cohesion

$$F_c = \frac{c}{c_m}$$

$$S_n = \frac{c}{\frac{c}{\gamma \cdot H}}$$

Factor of safety with respect to

H_b

$H_c \Rightarrow$ critical height

$$F_c = \frac{H_c}{H}$$

$$S_n = \frac{c}{\gamma H} = \frac{c_m}{\gamma H}$$

