Gauss $\rightarrow$ Divergence

$$\iint \underline{\mathbf{D}} . \underline{\mathbf{ds}} \quad = \iiint \rho \, dv \rightarrow \underline{\nabla} . \underline{\mathbf{D}} = \rho(\underline{\mathbf{r}})$$

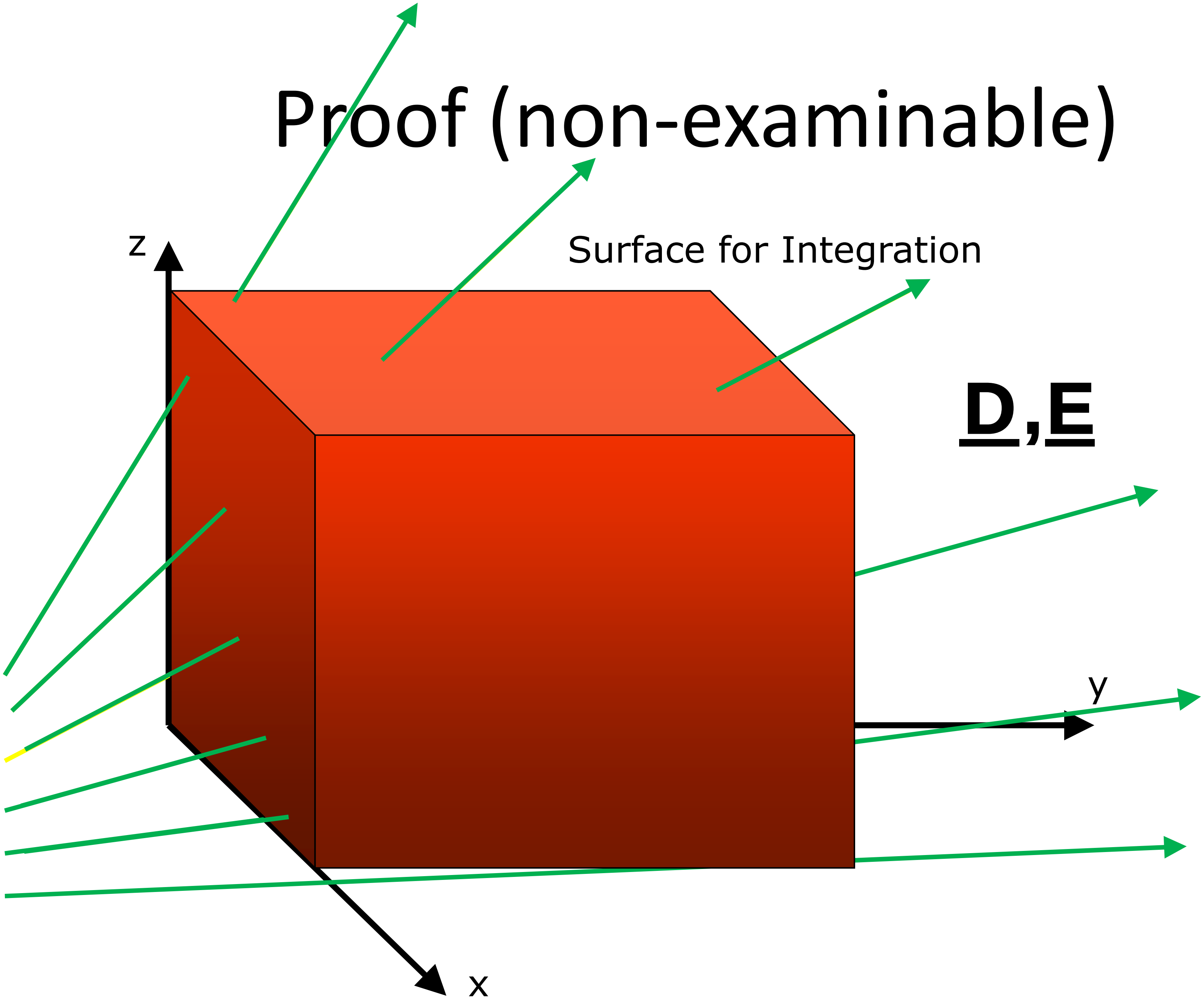
Divergence Theorem

Gauss→Divergence … why, oh why??

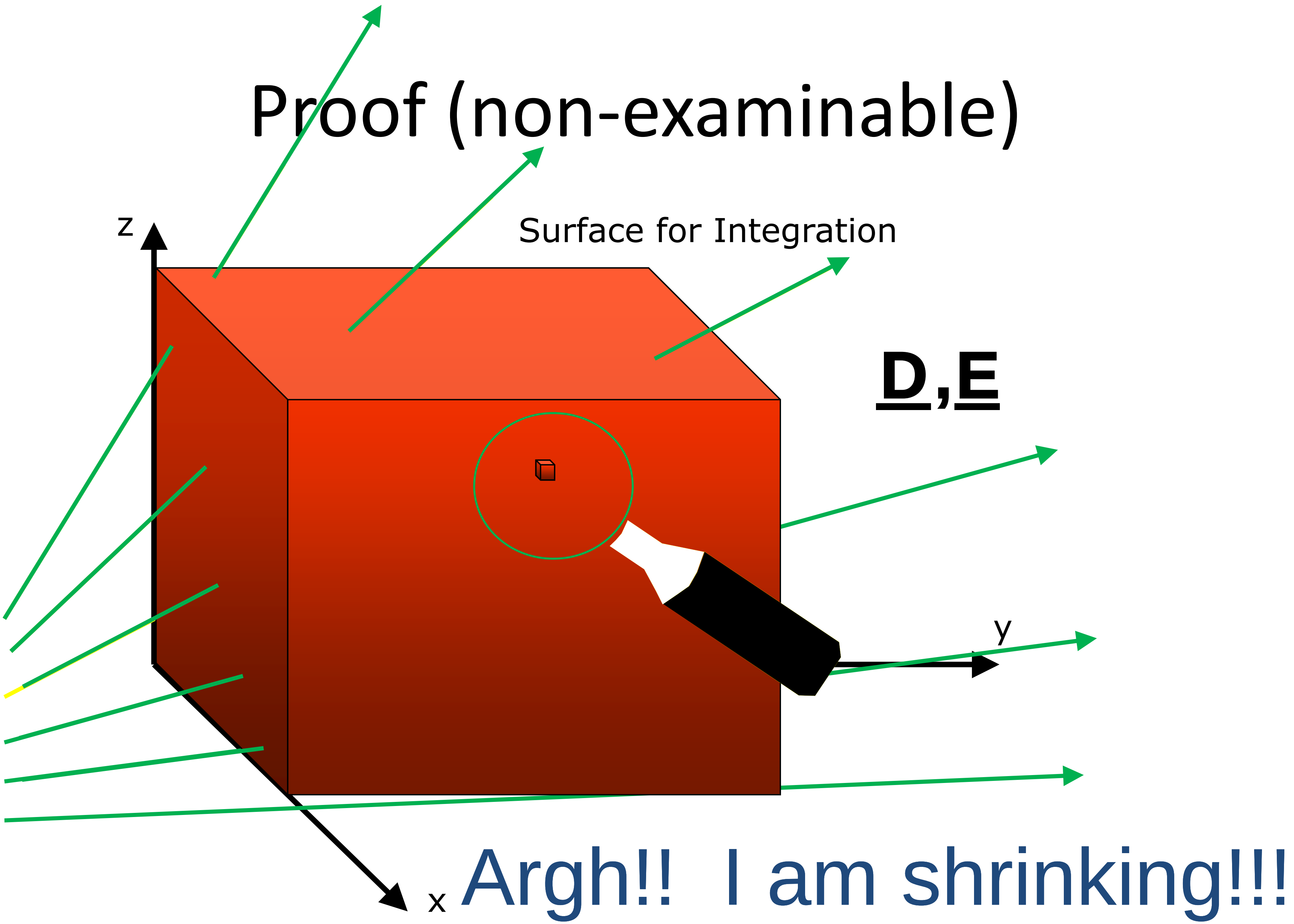
- $\iint \underline{\mathbf{D}}.\underline{\mathbf{ds}} = \iiint \rho(\underline{\mathbf{r}})dv$ is clearly a useful means of calculating $\underline{\mathbf{D}}$ and $\underline{\mathbf{E}}$ from a <u>macroscopic</u> (i.e.sizeable!) distribution of charge
  - It relates charge density in a volume of space to the field that it creates
- We want to find a relationship between charge density at a point $\rho(\underline{\mathbf{r}})$ and the fields $\underline{\mathbf{E}}(\underline{\mathbf{r}})$ and $\underline{\mathbf{D}}(\underline{\mathbf{r}})$ that it creates at that point

Cut to the chase

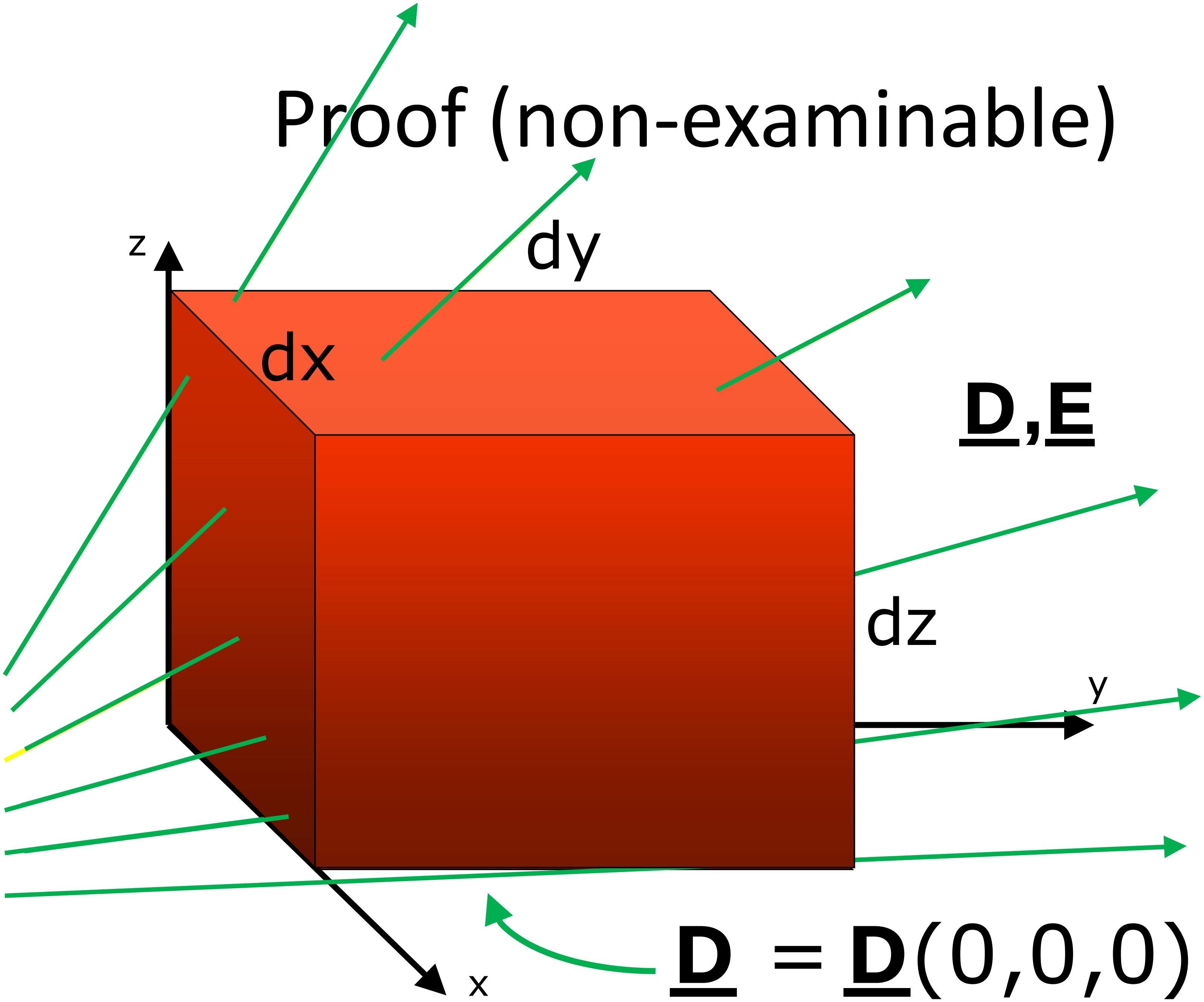
Proof (non-examinable)



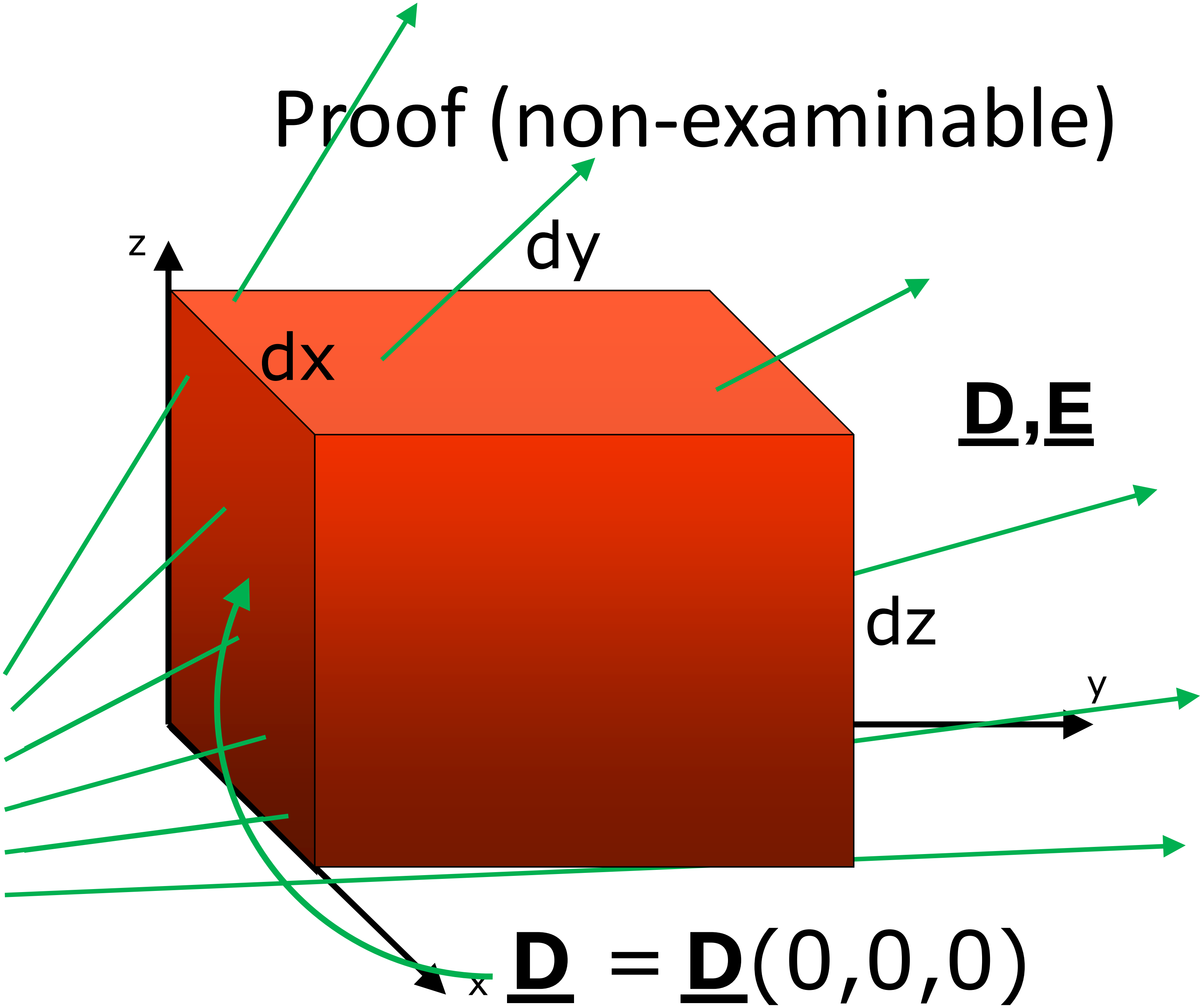
Proof (non-examinable)



Proof (non-examinable)

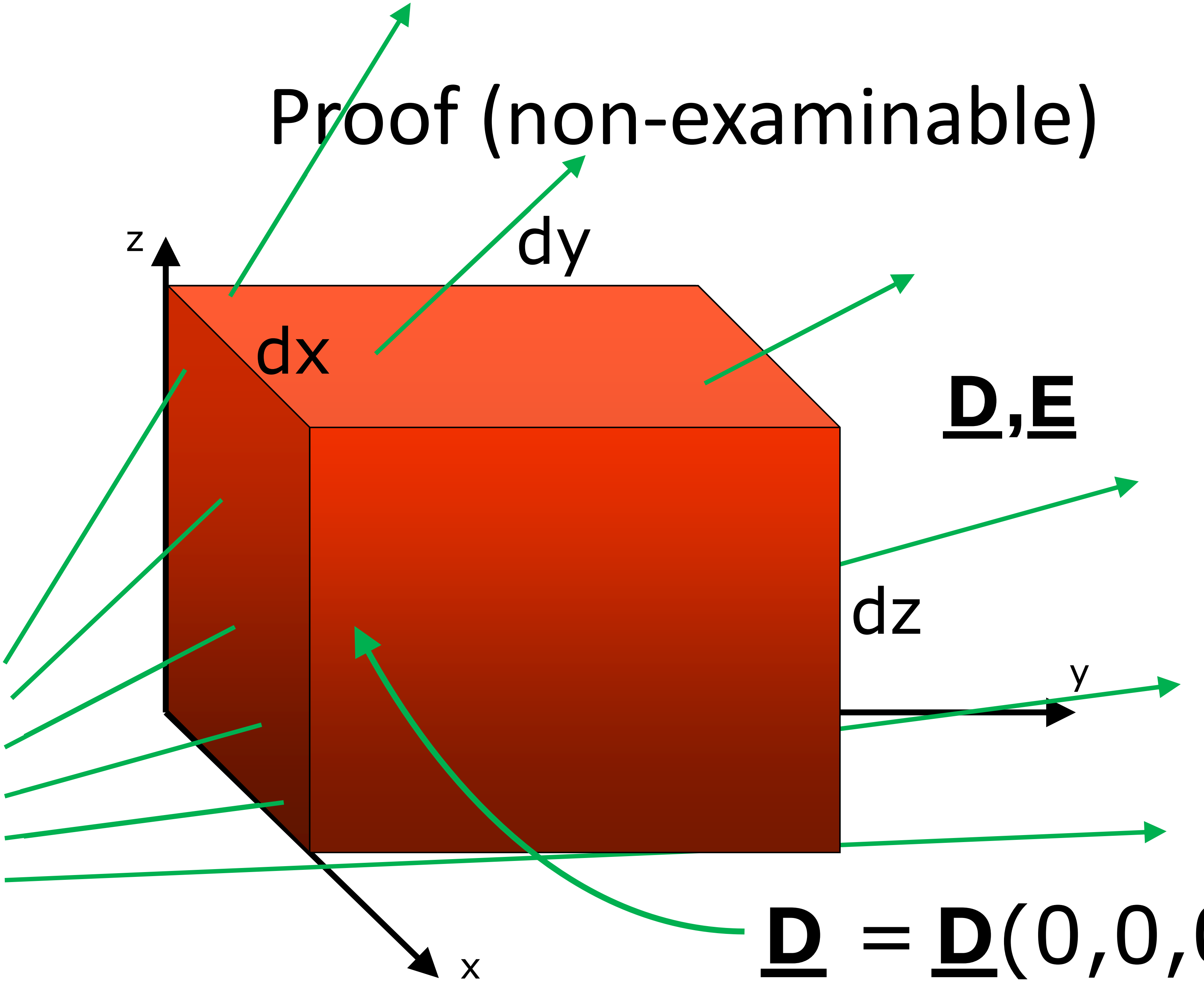


Proof (non-examinable)



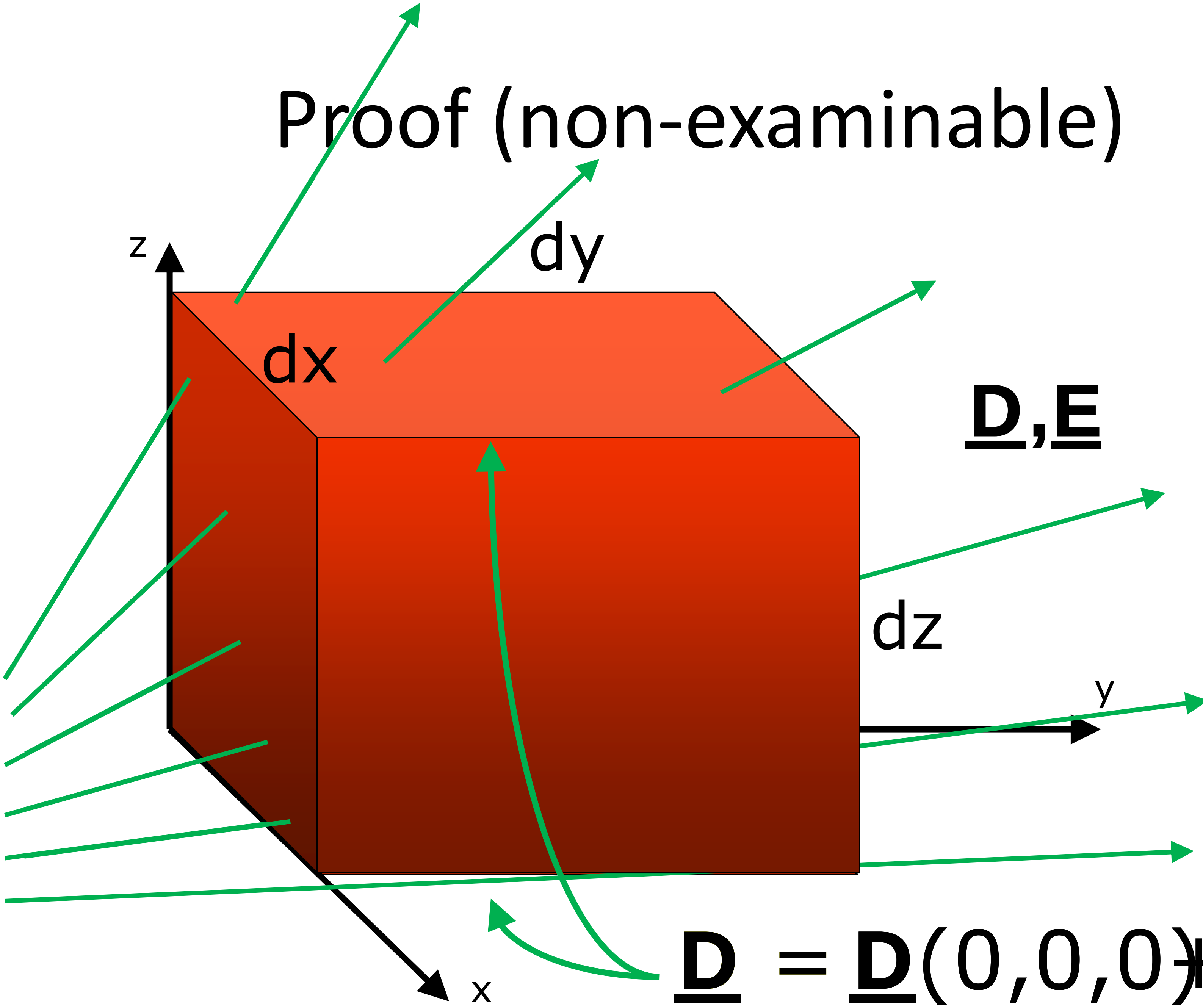
$$\underline{\mathbf{D}} = \underline{\mathbf{D}}(0,0,0)$$

Proof (non-examinable)



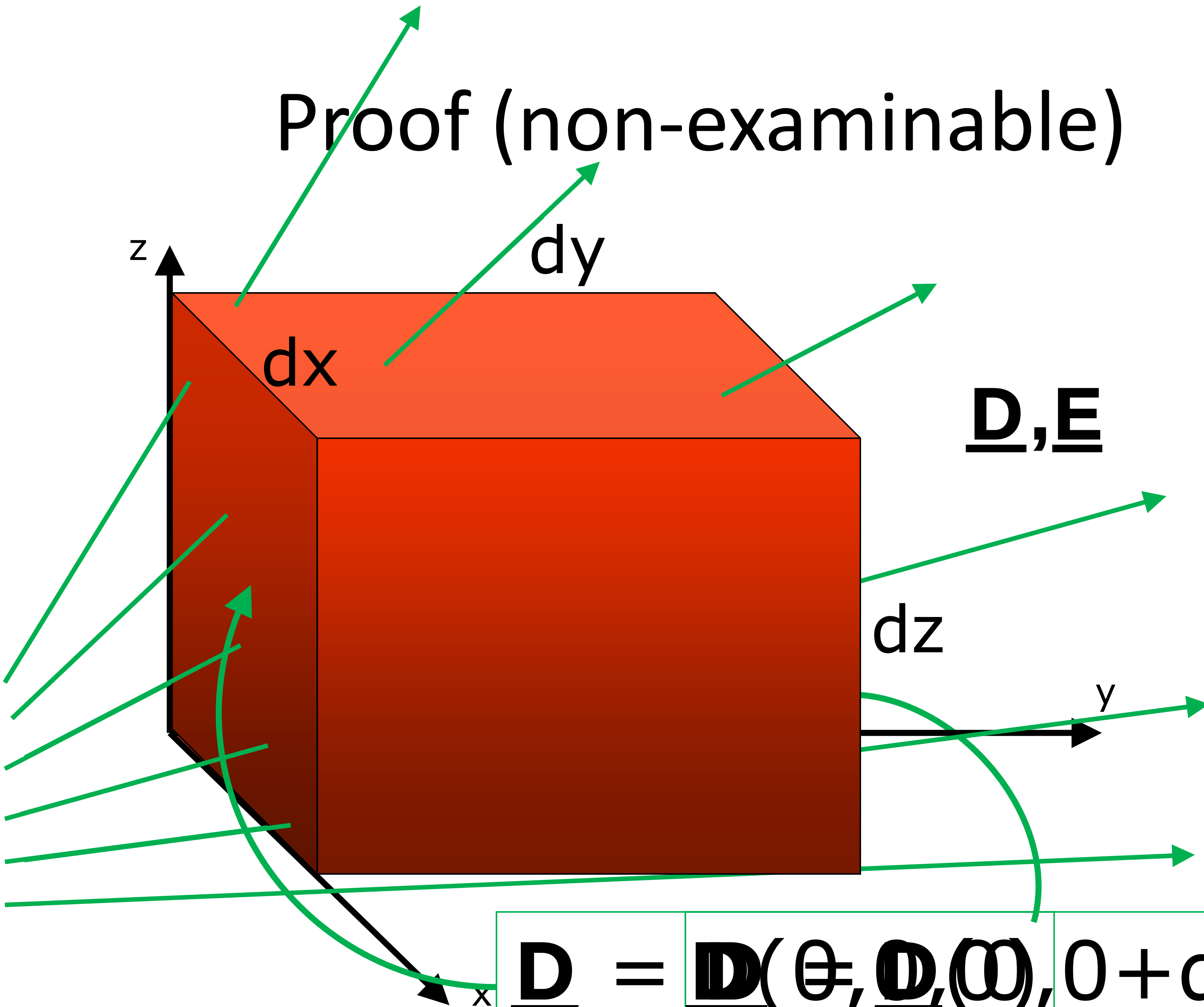
$$\underline{\mathbf{D}} = \underline{\mathbf{D}}(0,0,0)$$

Proof (non-examinable)



$\underline{\mathbf{D}} = \underline{\mathbf{D}}(0,0,0+dz)$

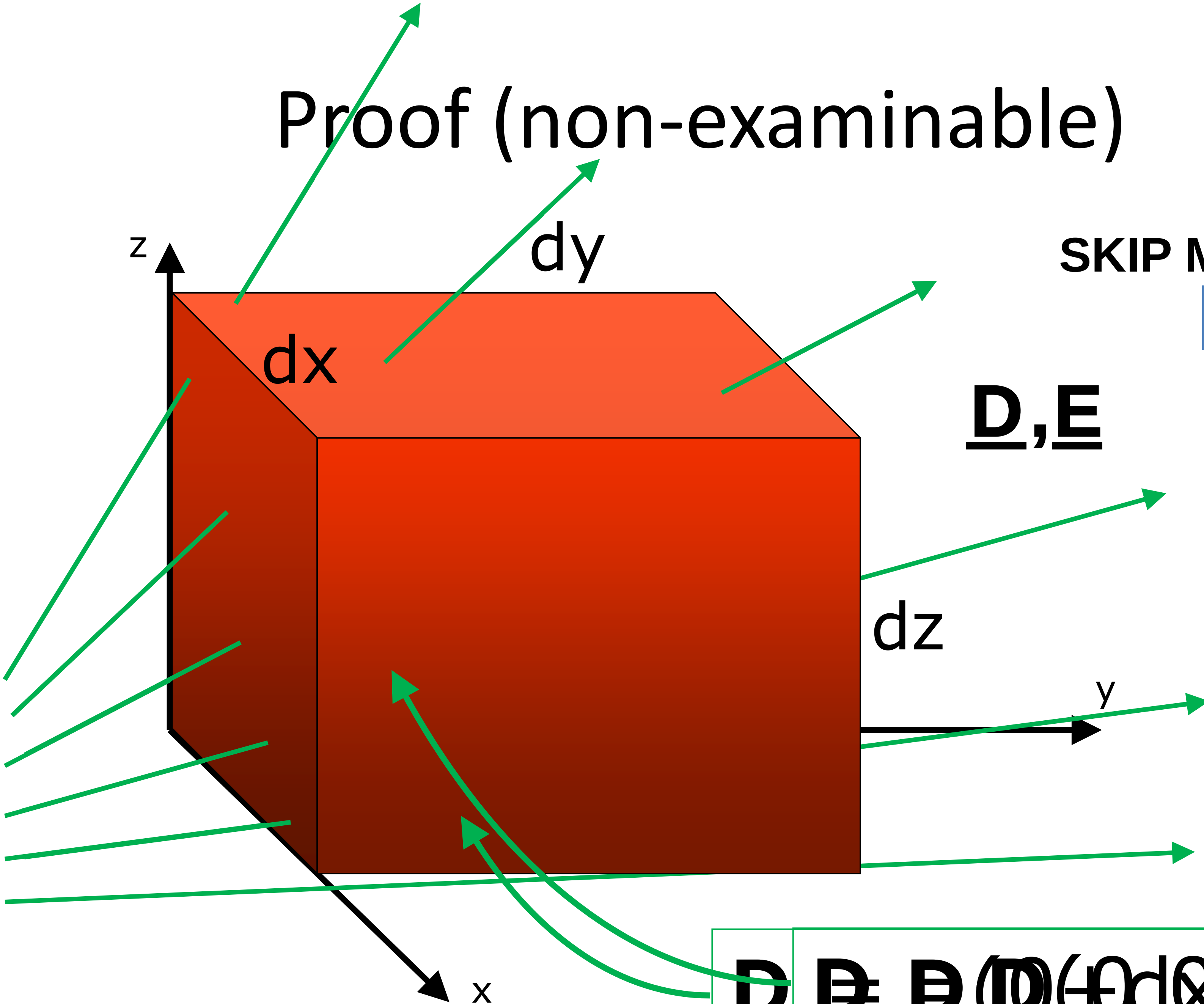
Proof (non-examinable)



$\underline{D} = \underline{D}(0,0,0)$

$= \underline{D}(0,0+dy,0)$

Proof (non-examinable)



SKIP MATHS

$\mathbf{D} = \mathbf{D}(0,0,0)$

$\mathbf{D} = \mathbf{D}(0+dx,0,0)$

Now the maths …

- Assume that, for example, $\underline{\mathbf{D}} = (D_x, D_y, D_z)$ over the entire left hand face, the back face and the bottom face … all the faces that meet at the origin
- $\underline{\mathbf{D}}$ is different on the other 3 faces
- Front face : $\underline{\mathbf{D}} = (D_{\mathbf{x+\Delta x}}, D_y, D_z)$
- Right face : $\underline{\mathbf{D}} = (D_x, D_{\mathbf{y+\Delta y}}, D_z)$
- Top face : $\underline{\mathbf{D}} = (D_x, D_y, D_{\mathbf{z+\Delta z}})$

Now the maths …

- Left face : $\underline{\mathbf{D}} = (D_x, D_y, D_z)$
  - $\underline{\mathbf{ds}}$ = (0, -dxdz, 0)
- Right face : $\underline{\mathbf{D}} = (D_x, D_{\mathbf{y+\Delta y}}, D_z)$
  - $\underline{\mathbf{ds}}$ = (0, +dxdz, 0)
- Bottom face : $\underline{\mathbf{D}} = (D_x, D_y, D_z)$
  - $\underline{\mathbf{ds}}$ = (0, 0, -dxdy)
- Top face : $\underline{\mathbf{D}} = (D_x, D_y, D_{\mathbf{z+\Delta z}})$
  - $\underline{\mathbf{ds}}$ = (0, 0, +dxdy)
- Back face : $\underline{\mathbf{D}} = (D_x, D_y, D_z)$
  - $\underline{\mathbf{ds}}$ = (-dydz, 0, 0)
- Front face : $\underline{\mathbf{D}} = (D_{\mathbf{x+\Delta x}}, D_y, D_z)$
  - $\underline{\mathbf{ds}}$ = (+dydz, 0, 0)

Now the maths …

- $\iint \underline{\mathbf{D}}.\underline{\mathbf{ds}}$ $= (D_x,D_y,D_z).(0, -dxdz, 0)$
  $+ (D_x,D_y,D_z).(0, 0, -dxdy)$
  $+ (D_x,D_y,D_z).(-dydz, 0, 0)$
  $+ (D_{\mathbf{x+\Delta x}},D_y,D_z).(dydz, 0, 0)$
  $+ (D_x,D_{\mathbf{y+\Delta y}},D_z).(0, dxdz, 0)$
  $+(D_x,D_y,D_{\mathbf{z+\Delta z}}).(0, 0, dxdy)$

Now the maths …

- $\iint \underline{\mathbf{D}}.\underline{\mathbf{ds}}$ $= -D_y dxdz - D_z dxdy - D_x dydz$
  $+(D_{x+\Delta x})dydz$
  $+(D_{y+\Delta y})dxdz$
  $+(D_{z+\Delta z})dxdy$
- $= -D_y dxdz - D_z dxdy - D_x dydz$
  $+(D_x + dx\partial D_x/\partial x)\ dydz$
  $+(D_y + dy\partial D_y/\partial y)dxdz$
  $+(D_z + dz\partial D_z/\partial z)dxdy$

Now the maths …

- $\iint \underline{\mathbf{D}}.\underline{\mathbf{ds}}$ $= (dx\partial D_x/\partial x)\, dydz$

  $+ (dy\partial D_y/\partial y)dxdz$

  $+ (dz\partial D_z/\partial z)dxdy$

  $= (\partial D_x/\partial x)\, dxdydz$

  $+ (\partial D_y/\partial y)dxdydz$

  $+ (\partial D_z/\partial z)dxdydz$

  $= (\partial D_x/\partial x)\, dv + (\partial D_y/\partial y)dv$

  $+ (\partial D_z/\partial z)dv$

- $\iint \underline{\mathbf{D}}.\underline{\mathbf{ds}}$ $= (\partial/\partial x, \partial/\partial y, \partial/\partial z).\, (D_x, D_y, D_z)\, dv$

$dxdydz = dv$

Now the maths …

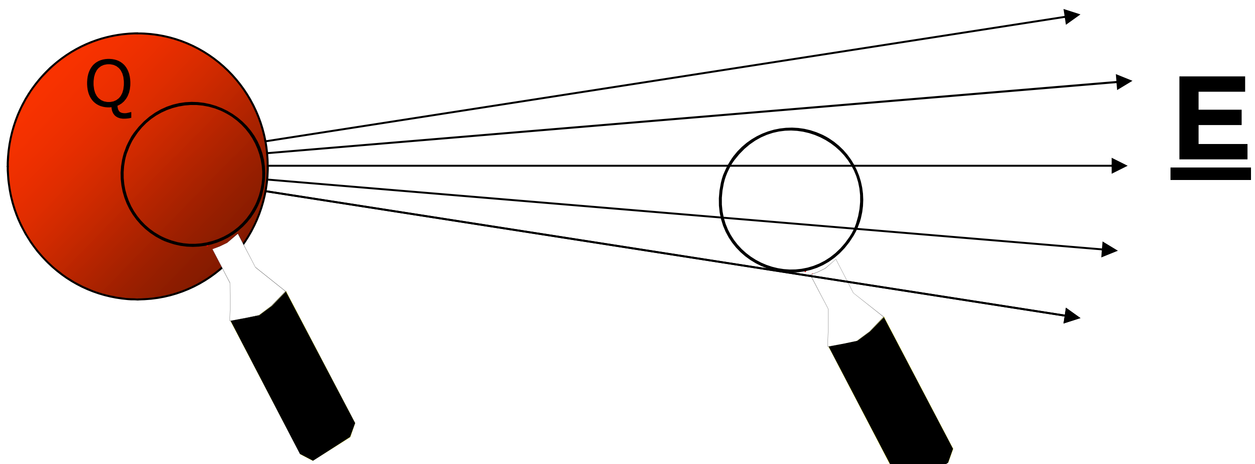
- $\iint \underline{\mathbf{D}}.\underline{\mathbf{ds}} = (\partial/\partial x, \partial/\partial y, \partial/\partial z).(D_x, D_y, D_z)dv$
- $\iint \underline{\mathbf{D}}.\underline{\mathbf{ds}} = \underline{\nabla}.\underline{\mathbf{D}}\, dv$ = charge enclosed
- $\underline{\nabla}.\underline{\mathbf{D}}dv = \iiint \rho\, dv$
  - for an infinitesimally small volume dv, ρ is constant
- $\underline{\nabla}.\underline{\mathbf{D}}dv = \rho \iiint dv = \rho dv$
- $\underline{\nabla}.\underline{\mathbf{D}} = \rho(\underline{\mathbf{r}})$
- This is the differential, or "at-a-point" version of Gauss's law, often called the Divergence Theorem
- $(\partial/\partial x, \partial/\partial y, \partial/\partial z) = \underline{\nabla}$ is the Divergence Operator

Gauss's Law/Divergence Theorem

- $\iint \underline{\mathbf{D}}.\underline{\mathbf{ds}} = \iiint \rho(\underline{\mathbf{r}})\, dv$
- $\underline{\nabla}.\underline{\mathbf{D}} = \rho(\underline{\mathbf{r}})$
- These are equivalent
- $\underline{\nabla} = (\partial/\partial x, \partial/\partial y, \partial/\partial z)$

So what does Divergence "mean"

- Not the same as "to diverge"

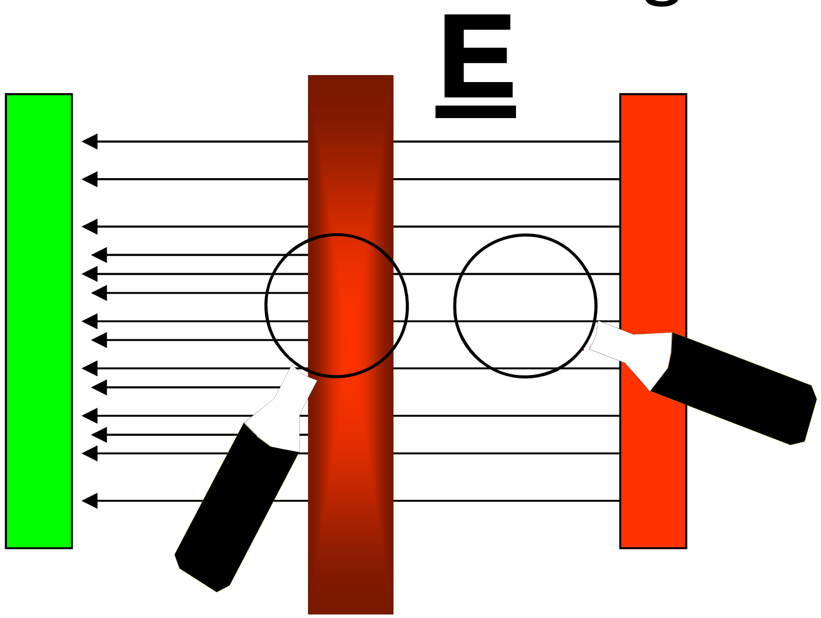


The electric field here diverges, and its DIVERGENCE is non-zero at this point, as there is charge present at the point, $\rho>0$.

The electric field here diverges, but its DIVERGENCE is zero at this point, as there is no charge present at the point, $\rho=0$.

So what does Divergence "mean"

- Not the same as "to diverge"



The electric field here does not diverge, and its DIVERGENCE is non-zero at this point, as there is charge present at the point, $\rho>0$.

The electric field does not diverge <u>and</u> its DIVERGENCE is zero at this point, as there is no charge present at the point, $\rho=0$.